A Practical Model to Approach Optimal Generation Maintenance Scheduling

Kevin Salgado, *Student Member, IEEE*, David Sebastián, *Senior Member, IEEE.*

***Abstract*—This paper shows a practical mathematical model to achieve an optimal generation maintenance scheduling considering a market environment. Proposed model achieves some of both GNECO and ISO's objectives, calculates an optimal scheduling plan, Units' Assignment (UA), and a forecast of power generations and power flows through a specified long-scale period. This model considers system constrains and uses multiple objective functions to minimize planning maintenance and operational costs just as minimize a given reliability index. Guaranteeing enough secure for each analyzed subperiod.**

Keywords—Generation Maintenance Scheduling (GMS), Mixed Integer Programming (MIP), Independent System Operator (ISO), per unit (p.u).

NOMENCLATURE

Variables:

$x_{i,j,k,t}$ Decision binary variable for the unit k of producer j connected to bus i in period t (1 if the unit k is on maintenance in period t and 0 otherwise).

$v_{i,j,k,t,s}$ Binary online status for unit k of producer j connected to bus i in period t and subperiod s (1 if unit k is online in subperiod s of period t and 0 otherwise).

$P_{i,j,k,t,s}$ Power generation unit k of producer j connected to bus i in period t and subperiod s.

$Fl_{i-j,t,s}$ Power flow of line $i-j$ in period t and subperiod s.

$\delta_{i,t,s}$ Angular offset of bus i in subperiod s of period t.

$IND_{t,s}$ Reliability index in period t and subperiod s.

$CO_{i,j,k,t,s}$ Operational cost of unit k owned by producer j connected to bus i in period t and subperiod s.

Parameters:

$abc_{i,j,k}$ Characteristics constants of the quadratic cost function of unit k of producer j connected to bus i.

$P_{i,j,k}^{min/max}$ Minimum/maximum power output (MW) of unit k of producer j connected to bus i.

$CM_{i,j,k}$ Maintenance cost for unit k of producer j in bus i.

$C_{pu}^{max/min}$ Maximum/minimum p.u maintenance costs allowed for any unit.

$D_{i,j,k}$ Maintenance duration in periods of unit k of producer j connected to bus i.

$D_{i,j,k1,k2}^{max}$ Constant equal to $\max\{D_{i,j,k1}, D_{i,j,k2}\}$.

$D_{i,j,k1,k2}^{min}$ Constant equal to $\min\{D_{i,j,k1}, D_{i,j,k2}\}$.

$Pi_{i,j,k}$ First period where unit k of producer j connected to bus i can begin its maintenance activity.

$Pf_{i,j,k}$ Last period where unit k of producer j connected to bus i can begin its maintenance activity.

XL_{i-j} Reactance in PU of line connected from bus i to bus j.

$Limit_{i-j}$ Maximum power flow (MW) allowed for branch $i-j$.

$SL_{t,s}$ Total system load in period t and subperiod s.

$LB_{i,t,s}$ Load in bus i for period t and subperiod s (percent of total system load).

FP_t Penalty factor in percent of any unit's maintenance cost for period t.

$Rmin_{t,s}$ Net minimum reserve (MW) in period t and subperiod s.

$N_{i,j,k1,k2}$ Number of separations in terms of time periods for the maintenance of units $k1$ and $k2$ of producer j connected to bus i.

$O_{i,j,k1,k2}$ Number of time periods during which the maintenance of units $k1$ and $k2$ of producer j connected to bus i should overlap.

Numbers:

I Number of nodes.

T Number of periods.

S Number of subperiods.

$Nmax_{j,t}$ Maximum number of units in maintenance owned by producer j in period t.

Sets:

$UNITS$ Set of indices of generation units $\{i,j,k\}$.

$lines$ Set of lines connected from bus i to bus j $\{i-j\}$.

Ω_j^E Set of pair of units of producer j that satisfy maintenance exclusion constrains.

Ω_j^P Set of pair of units of producer j that satisfy maintenance priority exclusion constrains.

Ω_j^S Set of pair of units of producer j that satisfy maintenance separation constrains.

Ω_j^O Set of pair of units of producer j that satisfy maintenance overlap constrains.

I. INTRODUCTION

GMS is tightly connected with power system planning and operation, the target of an optimal maintenance scheduling is to reduce operational and maintenance costs of each unit while satisfying system loads within an acceptable level of security. However, modern methodologies must consider a market environment meeting both ISO and GENCO's objectives, the first problem is the difference between those objectives, the ISO seeks to ensure a maximum security for the system while GENCO is trying to maximize its profits. Even when the ISO is the entity that must generate an optimal generation maintenance plan, is obligated to receive GENCO's petitions for their maintenance windows that satisfies GENCO's needs and must try to meet those petitions on the final maintenance scheduling plan.

There is a lot of research abording the GMS problem, nevertheless there is not a clear consensus about the best way to approach this problem because of its complexity. GMS is an integer, non-convex, stochastic problem, the extensive literature uses methods such as MIP, Genetic Algorithms, Benders Decomposition or even Artificial Intelligence to obtain an optimal GMS.

A. *Some important methodologies proposed on literature for GMS.*

In [1] and [2] are found the pioneer models for GMS using integer programming and MIP seeking an optimal plan for GMS, but those models are old and do not consider a lot of important constraints, anyway those are models for a centralized system.

A combination between Genetic Algorithm and Particle Swarm Optimization is used in [3] to approach a non-linear multi-objective GMS problem, however this model only guarantees local optimal solutions and ignores power generations and system constraints.

In [4] the ISO and GENCO's problems are resolved separate using decomposition methods (such as Dantzig-Wolfe and Benders decomposition) an then an algorithm is created in order to reach a convergence between ISO and GENCO's objective, but decompositions methods results in a more complicated procedure and higher difficulty convergence condition [5].

Reference [6] proposes a model for the ISO's optimal GMS and another one for the GENCO's optimal GMS, ISO's problem is solved first then GENCO's solution is compared (in terms of reliability) the algorithm adds penalty factors until GENCO's solution is similar enough to ISO's (in terms of reliability), but this method does not consider system's constraints.

In [7] a stochastic multi-objective model is used with a variant of the Normalized Normal Constraint (NNC) method called Augmented NNC or (ANNC) to obtain an optimal GMS, the Pareto frontier is used to mix the objectives functions, but again as [3] this model is non-linear and found solutions are only local.

In [8] a methodology is presented to reach the optimal GMS for the life-cycle of any unit, this model considers a market environment and explains the risks for not giving maintenance, and the loss of giving maintenance too early, however, this model works only for producers optimizing its profits.

B. *Aspects of the current work*

The methodology presented in this paper tries to reach an optimal level of reserve for each period which is ISO's objective while minimizing the maintenance costs of units (similar levels of p.u maintenance costs for every GENCO) which is GENCO's objective, besides guaranteeing the minimum operational cost through the planning period, considering system and GENCO's constraints.

The proposed procedure uses three different objectives but does not take the three of them simultaneously, instead turns the problem into three different but connected subproblems and solves each separately until desired convergence, below is described the basic functioning of the proposed algorithm.

Step 1) The GMS problem is solved considering only maintenance constraints and net minimum reserve, minimizing the variance (a non-convex function for this model) of a reliability index for each subperiod but also minimizing the maintenance costs which depends of GMS obtained plan, the result is a locally optimal solution that ensures an optimal level of security in each subperiod and guarantees lower maintenance costs, the algorithm checks 1) similar p.u maintenance costs to assure fairness among GENCO's, and 2) feasibility for all system constraints.

Step 2) The net minimum reserve is meeting in each subperiod thanks to Step 1, but it is necessary to minimize the amount of online units satisfying the load to reduce operational costs, so UA is solved in order to reduce operational costs, this is a convex integer problem and an optimal integer solution can be guaranteed.

Step 3) Once optimal units online variables have been found, the next step is to solve the economic dispatch in order to find a forecast of generations and power flows that reduces operational costs, this problem is completely convex, and an optimal solution can be guaranteed.

The literature mentioned before was important for the development of the work presented on this paper, for the unfamiliar reader, basic literature on electricity markets can be found in [9], [10] and [11].

II. MATHEMATICAL FORMULATION

The model consists of the three objective functions described below, the first one is composed by the variance of the reliability index and the average p.u maintenance costs as follows (1):

$$F1\colon \min\{var(IND_{t,s}) + aver(CMpu_{i,j,k,t})\} \qquad (1)$$

Where,

$$CMpu_{i,j,k,t} = \frac{\sum_{t=1}^{T} x_{i,j,k,t} CM_{i,j,k} FP_t}{D_{i,j,k} CM_{i,j,k}}$$

Represent unit maintenance cost in p.u of nominal maintenance cost given by the product of maintenance duration and maintenance costs for each unit ($D_{i,j,k}CM_{i,j,k}$), $var(.)$ represent the *variance* function and $aver(.)$ represent average function or the *arithmetic mean*. Note that $IND_{t,s}$ formula is represented as follows.

$$IND_{t,s} = \frac{\sum_{\{i,j,k\}\in UNITS}(1 - x_{i,j,k,t})Pmax(t,s)_{i,j,k} - SL_{t,s}}{\sum_{\{i,j,k\}\in UNITS} P_{i,j,k}^{max} - SL_{t,s}}$$

Note that $Pmax(t,s)_{i,j,k} \neq P_{i,j,k}^{max}$, first one is the real maximum output power of a unit in subperiod s of period t, and second one is the nominal maximum output power. This reliability index measures the degree of security throughout the days (subperiods) of the year and its formula is defined as the fraction between net reserve and gross reserve. Net reserve is the difference between the total system capacity considering units' availability and system load forecast in any subperiod and gross reserve is defined as the difference between sum of system nominal maximum capacity and the power forecast demand.

Since this reliability index measures security it is important to analyze the possible values of this variable, if $IND_{ts} = 1$ then there are not units' maintenance and system is in ideal conditions in subperiod s of period t so total load in this subperiod is being satisfied with all the possible reserve available, if $0 < IND < 1$ then a bad condition in subperiod s of period t exist, which means not all units are available (failure or maintenance) or even some units are not able to dispatch all of its nominal power. However, power demand is meeting within a certain amount of reserve (higher when $IND \to 1$). If $IND_{t,s} = 0$ then maintenance, failure or another bad condition exist, system load is ideally being satisfied but 1) there are no available reserves for subperiod s of period t, and 2) there not exists guarantee at all to satisfy system constraints since it depends completely of system capacity. Finally, if $IND_{t,s} < 0$ then maintenance, failure or another bad condition exist (even a higher demand than system capacity), and system load is not being satisfied for subperiod s of period t.

$F1$ is a quadratic non-linear function due to the function taken for $var(.)$, so it guarantees a local optimal GMS plan that minimizes maintenance costs while ensuring an optimal degree of security for each subperiod. The second objective function is optimal Units Assignment and is described in (2).

$$F2\text{: min} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{\{i,j,k\}\in UNITS} CAU_{i,j,k,t,s} \tag{2}$$

Where,

$$CAU_{i,j,k,t,s} = a_{i,j,k}P_{i,j,k,t,s}^2 + b_{i,j,k}P_{i,j,k,t,s} + v_{i,j,k,t,s}c_{i,j,k}$$

The target of $F2$ is to minimize the cost for having online units ($v_{i,j,k,t,s}$) to meet total demand in each subperiod, so (2) can find an optimal integer plan for the spinning reserve. The subproblem is a convex quadratic integer, and an optimal integer solution can be easily found.

The last objective function is described in (3) and minimizes the operational cost with a maintenance plan and the unit assignment variable ($v_{i,j,k,t,s}^*$) fixed, so it is now a convex quadratic problem guaranteeing an optimal global solution.

$$F3\text{: min} \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{\{i,j,k\}\in UNITS} v_{i,j,k,t,s}^* CO_{i,j,k,t,s} \tag{3}$$

Where

$$CO_{i,j,k,t,s} = a_{i,j,k}P_{i,j,k,t,s}^2 + b_{i,j,k}P_{i,j,k,t,s} + c_{i,j,k}$$

The set of constraints considered in this model are specified below as follows, (4)−(7) are system constraints, (8) − (17) are maintenance constraints, (18) is a logical constraint and (19) is a fairness constraint.

A. *Minimum reserve.*

Optimal GMS plan must satisfy system load while guaranteeing a minimum reserve in each subperiod, constraint (4.1) ensures that.

$$\sum_{\{i,j,k\}\in UNITS}(1 - x_{i,j,k,t})P_{i,j,k}^{max} - SL_{t,s} \geq Rmin_{t,s}, \quad \forall t, \forall s. \tag{4.1}$$

Where,

$$Rmin_{t,s} = \beta SL_{t,s}\frac{\sum_{t=1}^{T}\sum_{s=1}^{S}\left[\sum_{\{i,j,k\}\in UNITS} P_{i,j,k}^{max} - SL_{t,s}\right]}{\sum_{t=1}^{T}\sum_{s=1}^{S} SL_{t,s}}$$

β is a per unit constant ($0 < \beta < 1$) and the other two factors that multiply are 1) system load for subperiod s and period t and 2) a fraction made up of total gross reserve (the sum over periods and subperiods) divided by the total energy demanded. This minimum reserve parameter guarantees higher reserves in subperiods with higher demands, which is an appropriate criterion.

$Rmin$ can be considering as the non-spinning reserve, however considering the spinning reserve in the UA model is necessary to optimize operational costs. It can be defined a new spinning reserve parameter as $SR_{t,s} = \alpha Rmin_{t,s}$ where α is a per unit constant ($0 < \alpha < 1$). Constraint (4.2) considers the spinning reserve.

$$\sum_{\{i,j,k\}\in UNITS}(v_{i,j,k,t,s})P_{i,j,k}^{max} - SL_{t,s} \geq SR_{t,s}, \quad \forall t, \forall s. \tag{4.2}$$

B. *Load balance*

Constraint (5) ensures all power generated is being consumed, or all demand is being satisfied for each bus i in any period t and subperiod s.

$$\sum_{\{i,j,k\}\in UNITS} P_{i,j,k,t,s} + FL_{i-j,t,s} = LB_{i,t,s}, \forall i \in I, \forall t, \forall s \tag{5}$$

Where,

$$FL_{i-j,t,s} = \sum_{j=1}^{I} 100 * \frac{(\delta_{i,t,s} - \delta_{j,t,s})}{XL_{i-j}}$$

Note that $P_{i,j,k,t,s} = 0$; $\forall\{i,j,k\} \notin UNITS$ and $FL_{i-j,t,s} = 0$; $\forall\{i-j\} \notin lines$.

C. Power generation limits

Constraint (6) ensures power generation for each unit within its power limits for all periods and subperiods.

$$v_{i,j,k,t,s}P_{i,j,k}^{min} \le P_{i,j,k,t,s} \le v_{i,j,k,t,s}P_{i,j,k}^{max} \quad \forall\{i,j,k\} \in UNITS, \forall t, \forall s \tag{6}$$

D. Line-flow limits

Constraint (7) ensures power flows over lines within its operational limits for all periods and subperiods.

$$\left|FL_{i-j,t,s}\right| \le Limit_{i-j}, \forall\{i-j\} \in lines, \forall t, \forall s \tag{7}$$

E. Maintenance period

Constraint (8) ensures that each unit meets its number of maintenance periods.

$$\sum_{t=1}^{T} x_{i,j,k,t} = D_{i,j,k}, \forall\{i,j,k\} \in UNITS \tag{8}$$

F. Maintenance window

Constraint (9) ensures all maintenance activities are allocated within its maintenance window for each unit.

$$\sum_{t=Pi_{i,j,k}}^{Pf_{i,j,k}} x_{i,j,k,t} = D_{i,j,k}, \forall\{i,j,k\} \in UNITS \tag{9}$$

G. Continuous maintenance

Constraint (10) ensures once maintenance has started it does not end until the periods duration has been completed.

$$x_{i,j,k,t} - x_{i,j,k,t-1} \le x_{i,j,k,(t+D_{i,j}-1)}, \quad \forall\{i,j,k\} \in UNITS, \forall t. \tag{10}$$

Note that $x_{i,j,k,t} = 0;\ \forall t \le 0, \forall t > T$.

H. Maximum number of units owned by a producer that can be simultaneously in maintenance

Constraint (11) below ensures a limit of units that producer i can have in maintenance in period t.

$$\sum_{\{i,j,k\} \in UNITS} x_{i,j,k,t} \le Nmax_{j,t}, \qquad \forall j \in NPr. \tag{11}$$

Where NPr is the set of total producers, note that $x_{i,j,k,t} = 0;\ \forall\{i,j,k\} \notin UNITS$.

I. Maintenance priority

Constraint (12) below, enforces priority in maintenance for units $k1$ and $k2$ both owned by producer i.

$$\sum_{\tau=1}^{t} x_{i,j,k1,\tau-1} - x_{i,j,k2,t} \ge 0, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^P, \forall t. \tag{12}$$

Note that $x_{i,j,k,t} = 0;\ \forall t \le 0, \forall\{i,j,k\} \notin UNITS$.

J. Maintenance exclusion

Constraint (13) does not allow simultaneous maintenance between two specified units owned by the same producer.

$$x_{i,j,k1,t} + x_{i,j,k2,t} \le 1, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^E, \forall t. \tag{13}$$

Please note that $x_{i,j,k,t} = 0;\ \forall\{i,j,k\} \notin UNITS$.

K. Separation between maintenances

This constraint is divided into two, (14) and (15) enforce the separation within a specified number of periods for maintenance outage of units $k1$ and $k2$ (both owned by producer j).

$$\sum_{\tau=1}^{t} x_{i,j,k1,(\tau-D_{i,j,k1}-N_{i,j,k1,k2})} - x_{i,j,k2,t} \ge 0, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^E, \forall t. \tag{14}$$

$$\sum_{\tau=1}^{t} D_{i,j,k1,k2}^{min} x_{i,j,k1(\tau-D_{i,j,k1}-N_{i,j,k1,k2})} - \sum_{\tau=1}^{t} D_{i,j,k1,k2}^{max}\, x_{i,j,k,\tau} \le 0, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^E, \forall t. \tag{15}$$

Note that $x_{i,j,k,t} = 0;\ \forall t \le 0, \forall\{i,j,k\} \notin UNIT$, besides unit $k1$ is going to start its maintenance first, then $k2$.

L. Overlap in maintenance

Just as in the *separation between maintenance*, this constraint is divided into (16) and (17), those constraints ensure that maintenance of units $k1$ and $k2$ (both owned by producer j) overlap a specified number of periods.

$$\sum_{\tau=1}^{t} x_{i,j,k1,(\tau-D_{i,j,k1}+O_{i,j,k1,k2})} - x_{i,j,k2,t} \ge 0, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^O, \forall t. \tag{16}$$

$$\sum_{\tau=1}^{t} D_{i,j,k1,k2}^{min} x_{i,j,k1(\tau-D_{i,j,k1}+O_{i,j,k1,k2})} - \sum_{\tau=1}^{t} D_{i,j,k1,k2}^{max}\, x_{i,j,k,\tau} \le 0, \quad \forall i, \forall j, \forall\{k1,k2\} \in \Omega_j^O, \forall t. \tag{17}$$

Note that $x_{i,j,k,t} = 0;\ \forall t \le 0, \forall\{i,j,k\} \notin UNIT$, besides, unit $k1$ is going to start its maintenance first, then $k2$.

M. Logical online constraint

Constraint (18) below is a logical constraint for the status of each unit, it ensures that the unit i of producer j connected to bus i cannot be online while maintenance activity exists in any period t.

$$\left(1 - x_{i,j,k,t}\right) \ge v_{i,j,k,t,s}, \quad \forall\{i,j,k\} \in UNITS, \forall t, \forall s. \tag{18}$$

N. Fairness among GENCOs' maintenance costs

Constraint (19) below assures similar level of p.u maintenance costs for all unit owners which is an important goal for a decentralized system.

$$C_{pu}^{min} \le CMpu_{i,j,k,t} \le C_{pu}^{max}, \forall\{i,j,k\} \in UNITS, \forall t. \quad (19)$$

To apply this constraint, it is necessary to set the values of $C_{pu}^{max/min}$ to define the gap $G = C_{pu}^{max} - C_{pu}^{min}$, if the gap tends to zero optimal solution will be better but harder to find.

III. PROPOSED PROCEDURE

The non-linear $F1$ subproblem is solved first, considering constraints (4.1), (8)−(17), and (19). The result is a local optimal solution called $F1S_q$ (iteration $q = 0$) from now on, found variables are $x_{i,j,k,t}^*$ and $IND_{t,s}^*$, the next step is to fix those found variables and consider only constraints (5)−(7) and (18) to see if $x_{i,j,k,t}^*$ and $IND_{t,s}^*$ are feasible solutions for all system constraints, if they are not feasible, then add constraint (20) to solve the problem again and obtain $F1S_{q+1}$.

$$IND_{t,s}^{q+1} > \min\{IND_{t,s}^{q}\}, \qquad \forall t, s \quad (20)$$

Infeasibility means a lack of units to satisfy the system dispatch at a certain subperiod s of period t. It is easy to know that period with less available units is the one with lowest IND value. Constraint (20) above ensures to find another local optimal solution with higher IND values at the $q+1$ iteration, this procedure repeats until the algorithm finds a feasible local optimal solution at the q iteration $F1S_q$ ($x_{i,j,k,t}^*$ and $IND_{t,s}^*$).

In order to minimize total operational costs, the second step is to optimize UA, the algorithm drop $F1$ (with feasible $x_{i,j,k,t}^*$ and $IND_{t,s}^*$ fixed) and takes $F2$, with maintenance variables fixed, the algorithm only considers system constraints (4.2)−(7) and the logical constraint (18) in order to find the optimal status $v_{i,j,k,t,s}^*$ of each unit for all periods and subperiods.

Once an optimal $v_{i,j,k,t,s}^*$ has been found, the last step is to fix this variable, drop $F2$ and consider $F3$ as the objective now, the algorithm drops logical constraint (18) and now the problem is completely convex, $F3$ is a convex quadratic polynomial, and the constrains considered (5)−(7) are lineal, so an optimal solution can be found.

This proposed procedure was developed in AMPL (A Mathematical Programming Language), a high-level modeling language [12] as follows (see $Fig. 1$).

The first subproblem $F1$ was computed using the solver XPRESS with its integer quadratic non-convex option [13] a Branch and Bound (B&B) model was approached (see *Fig. 1*) to find a local optimal solution and a Quadratic Programming (QP) model to check feasibility. Second subproblem (Units Assignment) was computed using the CPLEX solver [14] and a simplex Mixed Integer Programing (MIP) model among with a B&B model were approached, note that this problem is an integer quadratic convex one. Finally, last subproblem $F3$ was aborded using the CPLEX solver, a QP model solves this last subproblem since it is a convex quadratic one in continuous variables.

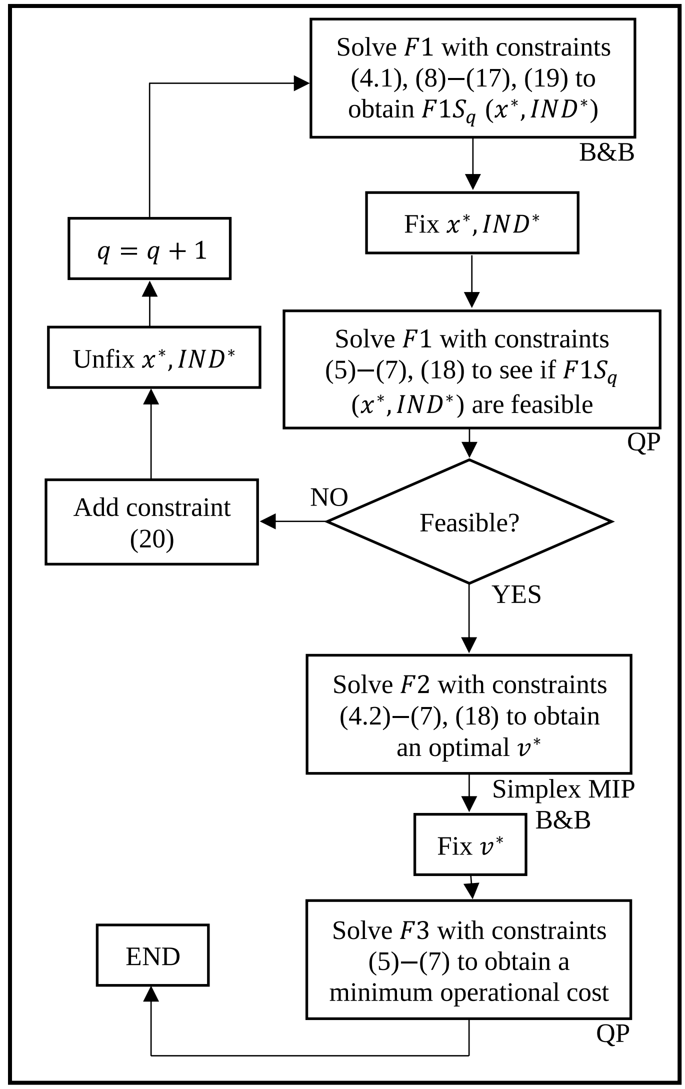

Figure 1. Algorithm flow chart.

IV. CASE STUDY

IEEE-RTS 24 bus system is tested with this proposed method. This system is made of 32 generating units, 20 demand sides, 24 buses and 38 transmission lines. Penalty Factors $FP_{i,j,k,t}$ are proposed with following criteria: higher penalty factors for higher load demand periods. The generating units encompass 14 GENCOs and a one year study period is considered within 52 periods (weeks of the year) and 7 subperiods (days of the week), for detailed IEEE-RTS 24 bus system information please check [15]. $\alpha = 1$ and $\beta = 0.1$ are considered. Two different cases are tested to compare different objective functions for subproblem $F1$.

Case 1: $F1: \min \sum_{\{i,jk\} \in UNITS} \sum_{t=1}^{T} x_{i,j,k,t} CM_{i,j,k} FP_t$ is the objective function for first case, this function finds the minimum feasible maintenance cost for all units.

Case 2: $F1: \min var(IND_{t,s}) + aver(CMpu_{i,j,k,t})$, this is the proposed model original objective function, finds a low maintenance cost and a well maintenance distribution.

Table 1. Comparation between results of the two cases.

Case	Number of iterations q	Total Maintenance Cost ($\$10^3$)	Total Operational Cost ($\$10^6$)		Index Variance
			Without UA	With UA	
1	4	52.8578	10.6478*	10.1078	0.06528
2	1	55.2029	10.4984*	10.0424	0.01766

*These results were obtained without considering $F2$ and $F3$.

Table 1 shows important comparation between the two cases, four iterations were needed to find a feasible solution for case 1 because in this case problem $F1$ allocate maintenance of units in low penalty factor (FP_t) periods, this may result in many unavailable units for certain periods the consequence is the infeasibility of $F2$ and $F3$ problems, however when a feasible solution is found minimum operational cost is conditioned in periods with higher number of unavailable units. On the other hand, case 2 reached convergence with the first iteration because $F1$ guarantees a well distribution level of maintenance activities, so offline units are dispersed throughout periods and subperiods (note that lower demand periods reach higher number of maintenances). The advantage of Case 1 is a lower total maintenance cost, but Case 2 shows lower operational costs, higher levels of security, and greater computational robustness, which makes Case 2 objective function better than Case 1.

Table 2 gives an important comparation between each unit operational cost for both cases. Case 2 total operational cost is lower than Case 1 because of the higher number of available units to meet the load in periods with higher demand.

Table 2. Operational Cost results, both cases.

GENCO	Units	Operational Cost ($\$10^4$)	
		Case 1	Case 2
1	1, 2	31.220, 33,066	32.775, 34.2992
2	3, 4	0.0, 0.054	0.0, 0.0
3	5, 6	32.774, 32.108	34.0559, 33.4432
4	7, 8	0.0, 0.0	0.0, 0.0
5	9, 10, 11	26.083, 0.068, 9.550	27.805, 0.137, 8.424
6	12, 13, 14, 15	3.339, 1.115, 44.584, 63.413	0.8374, 0.4579, 47.9559, 63.7229
7	16, 17, 18, 19, 20	1.4152, 1.0444, 0.718, 0.4718, 0.1615	1.380, 1.049, 0.7426, 0.482, 0.219
8	21	63.4435	63.6755
9	22	63.4335	63.6900
10	23	116.475	116.475
11	24	116.730	116.7
12	25, 26, 27, 28, 29, 30	61.968, 26.635, 18.53, 8.729, 5.111, 47.998	64.06, 31.24, 13.58, 2.135, 0.0, 44.529
13	31	63.3164	63.849
14	32	136.1914	136.471
Total Operational Cost ($\$10^4$)		1010.78	1004.24

Table 3. Maintenance results, case 1

GENCO	Units	Capacity (MW)	Weeks on Maintenance	Maintenance Cost (p.u)
1	1, 2	76, 76	27-29, 27-29	0.74, 0.74
2	3, 4	20, 20	26-27, 13-14	0.74, 0.80
3	5, 6	76, 76	27-29, 27-29	0.74, 0.74
4	7, 8	20, 20	26-27, 30-31	0.74, 0.80
5	9, 10, 11	100, 100, 100	26-28, 26-28, 26-28	0.75, 0.75, 0.75
6	12, 13, 14, 15	100, 100, 100, 155	27-29, 25-27, 26-28, 29-32	0.75, 0.77, 0.75, 0.79
7	16, 17, 18, 19, 20	12, 12, 12, 12, 12	27-28, 27-28, 27-28, 27-28, 27-28	0.73, 0.73, 0.73, 0.73, 0.73
8	21	155	25-28	0.77
9	22	155	29-32	0.79
10	23	400	10-15	0.80
11	24	400	10-15	0.80
12	25, 26, 27, 28, 29, 30	197, 197, 197, 197, 197, 197	12-15, 10-13, 25-28, 12-15, 29-32, 29-32	0.80, 0.80, 0.77, 0.80, 0.78, 0.78
13	31	155	29-32	0.79
14	32	350	7-11	0.84
Average Units Maintenance Cost (p.u)				0.7668

Table 4. Maintenance results, case 2.

GENCO	Units	Capacity (MW)	Weeks on Maintenance	Maintenance Cost (p.u)
1	1, 2	76, 76	26-28, 27-29	0.75, 0.74
2	3, 4	20, 20	27-28, 27-28	0.73, 0.73
3	5, 6	76, 76	27-29, 29-31	0.74, 0.78
4	7, 8	20, 20	27-28, 27-28	0.73, 0.73
5	9, 10, 11	100, 100, 100	30-32, 26-28, 26-28	0.80, 0.75, 0.75
6	12, 13, 14, 15	100, 100, 100, 155	27-29, 27-29, 27-29, 12-15	0.74, 0.74, 0.74, 0.80
7	16, 17, 18, 19, 20	12, 12, 12, 12, 12	27-28, 27-28, 27-28, 27-28, 27-28	0.73, 0.73, 0.73, 0.73, 0.73
8	21	155	14-17	0.83
9	22	155	13-16	0.82
10	23	400	20-25	1.00
11	24	400	35-40	0.91
12	25, 26, 27, 28, 29, 30	197, 197, 197, 197, 197, 197	14-17, 10-13, 10-13, 9-12, 29-32, 18-21	0.82, 0.80, 0.80, 0.82, 0.78, 0.99
13	31	155	31-34	0.86
14	32	350	1-5	0.92
Average Units Maintenance Cost (p.u)				0.7909

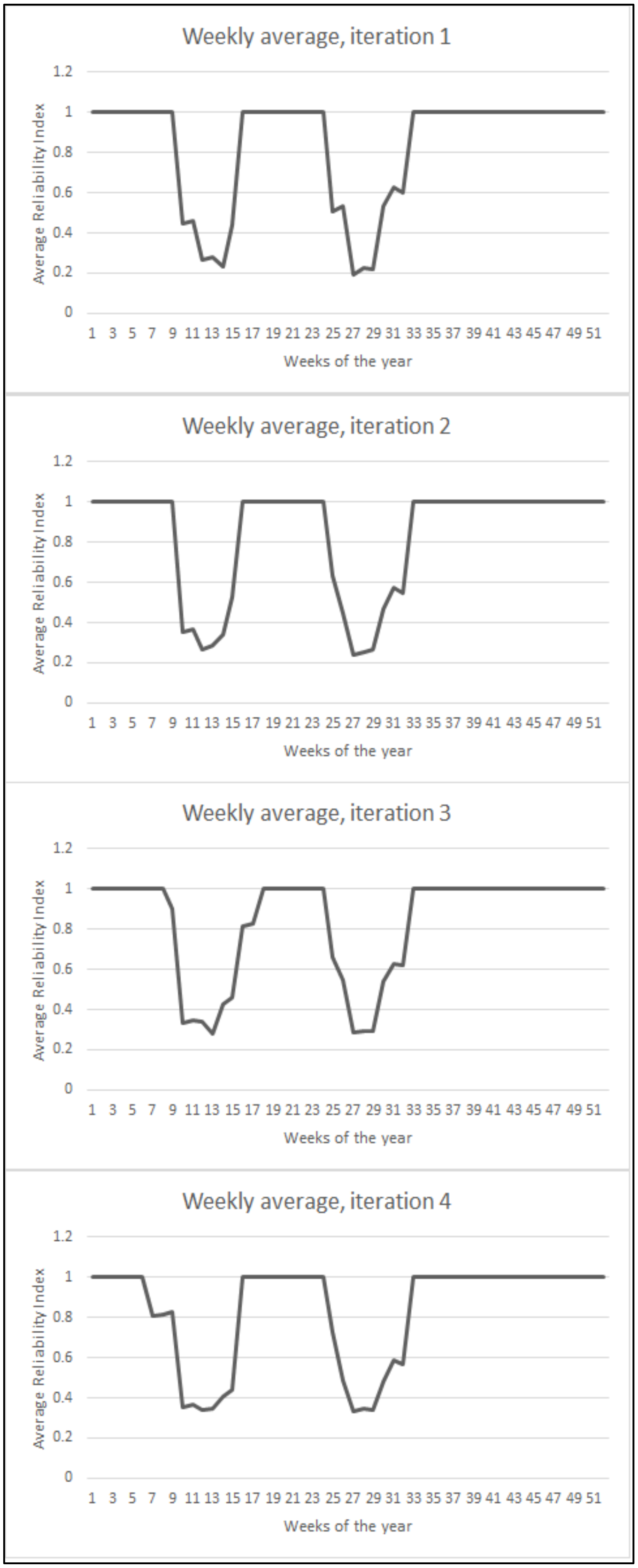

Figure 2. Evolution of reliability index, case 1.

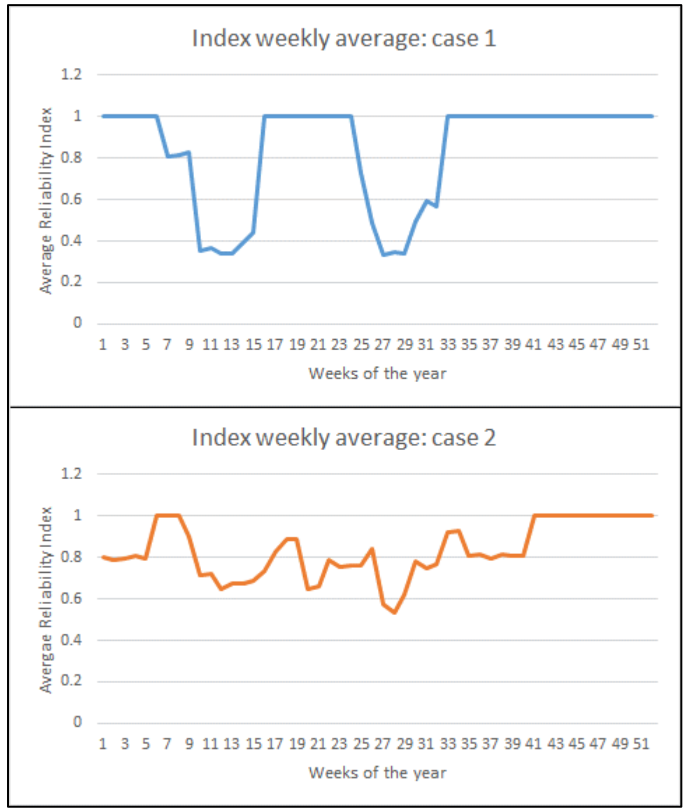

Figure 3. Comparation between cases 1 and 2 reliability indexes.

Tables 3 and *4* give summary information about maintenance results for cases 1 and 2 respectively, since case 1 seeks to minimize total maintenance cost without considering variance, maintenance cost is generally lower than case 2 for each unit, but *Fig. 3* shows a comparation between average reliability index in each period for both cases, where case 1 have two periods with a low reliability index (about 0.3) because those periods have lower maintenance cost, meanwhile case 2 maintain reliability index under an acceptable level any period of the year. Since case 1 needed 4 iterations to reach a feasible solution, *Fig. 2* shows the evolution of reliability index for each iteration until a feasible solution was found, low reliability index between weeks 9-15 and weeks 25-33 must change to find feasibility and satisfy all system constraints.

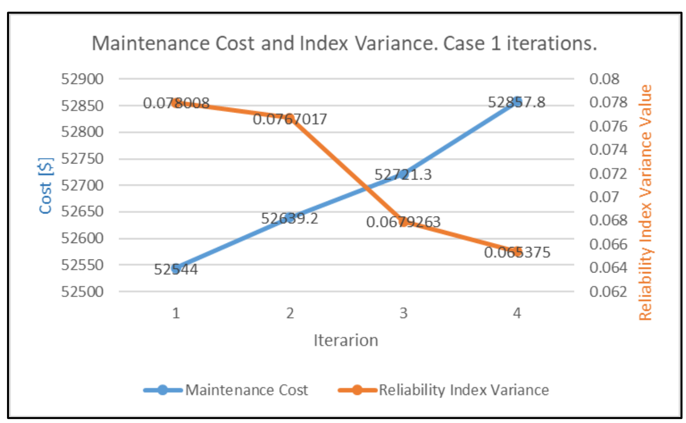

Figure 4. Evolution of maintenance cost and reliability index variance, case 1.

Maintenance costs and reliability index variance change in each case 1 iteration, *Fig. 4* shows the evolution of total maintenance cost and the evolution of the variance of reliability index for each iteration until a feasible solution was found. Total maintenance cost was forced to rise, and variance had to reduce its value to find a feasible solution in iteration number four.

The model is implemented for both cases under AMPL [12] on a computer equipped with Intel CORE i5, running at 2.30GHz with 8GB of RAM. It took 108.18 seconds to solve case 1 subproblems and 86.75 seconds to solve case 2 ones.

V. CONCLUSIONS

The model proposed in this paper finds an optimal GMS plan within low maintenance costs, ensuring a well level of system security for all periods and subperiods, besides this model minimizes operational costs through Units Assignment optimization and shows power generations and power flows forecast throughout analyzed subperiods with a lineal economic dispatch. This procedure is simple to implement in practice, requires a small amount of computing time and requires basic data management. Constraints $(8)-(17)$ allows producer to allocate its units maintenance on preferred periods, which is appropriate in deregulated systems.

ACKNOWLEDGMENTS

Authors thanks to the Istituto Politécnico Nacional (IPN), to the Sección de Estudios de Posgrado e Investigación ESIME Zacatenco and to CONACyT for the help provided.

VI. REFERENCES

[1] J. F. Dopazo y H. M. Merril, «Optimal Generation Maintenance Scheduling Using Integer Programming,» *IEEE Transaction on Power Systems,* pp. 1537 - 1545, 1975.

[2] D. Chattopadhyay, K. Bhattacharay y J. Parikh, «a system Approach to Least-Cost Maintenance Scheduling for an Interconnected Power System,» *IEEE Transactions on Power Systems,* vol. 10, nº 4, pp. 2002-2007, 1995.

[3] J. Shu, L. Zhang, H. Bing y H. Xianchao, «global Generator and Transmission Maintenance Scheduling Based On a Mixed Intelligent Optimal Algorithm in Power Market,» *IEEE,* pp. 1-5, 2006.

[4] M. Shahidehpour y M. K. C. Marwali, Maintenance Scheduling in Restructured Power Systems, New York: Springer, 2000.

[5] M. Shahidehpour y M. K. C. Marwali, «Long-term Transmission And Generation Maintenance Scheduling with Network, Fuel And Emission Constraints,» *IEEE Transactions on Power Systems,* vol. 14, nº 3, 1999.

[6] A. J. Conejo, R. García-Bertrand y M. Díaz-Salazar, «Generation Maintenance Scheduling in Restructured Power Systems,» *IEEE Transactions on Power Systems,* vol. 20, nº 2, pp. 984-992, 2005.

[7] B. Bagheri y N. Amjady, «Stochastic Multiobjective Generation Maintenance Scheduling using Augmented Normalized Normal Constraint Method and Stochastic Decision Maker,» *Wiley Int. Trans. Electr. Energ. Syst.,* nº 29, pp. 1 - 20, 2019.

[8] D. Chattopadhyay, «Life-Cycle Maintenance Management of Generating Units in a Competitive Enviroment,» *IEEE Transactions on Power Systems,* vol. 19, nº 2, pp. 1181 - 1189, 2004.

[9] A. J. Wood y F. Wollenberg, «Power Generation, Operation and Control,» *Wiley New York,* vol. 2, 2014.

[10] M. Shahidehpour, H. Yamin y Z. Li, Market Operations in Electric Power Systems, New York: IEEE, Wiley, 2002.

[11] M. Shahidehpour y M. Alomoush, Restructured Electrical Power Systems Operation, Trading and Volatility, New York: Marcel Dekker, 2001.

[12] Baytech Web Design, «AMPL,» AMPL Optimization Inc., 2020. [En línea]. Available: https://ampl.com/. [Último acceso: 21 March 2020].

[13] Baytech Web Design, «Xpress Options for AMPL,» AMPL Optimization Inc., 2020. [En línea]. Available: https://ampl.com/products/solvers/solvers-we-sell/xpress/options/. [Último acceso: 21 March 2020].

[14] Baytech Web Design, «CPLEX for AMPL,» AMPL Optimization Inc., 2020. [En línea]. Available: https://ampl.com/products/solvers/solvers-we-sell/cplex/. [Último acceso: 21 March 2020].

[15] P.M. Subcommitiee, «IEEE Reliability Test System,» *IEEE Transactions on Power Apparatus and Systems,* vol. PAS 98, nº 6, pp. 2047-2054, Nov. 1979.

VII. BIOGRAPHIES

Kevin Jomeini Salgado Carrillo
Received the Ingeniero Electricista degree from the Instituto Politécnico Nacional, CDMX, Mexico in 2019. He is currently working toward the master's degree in Electrical Power Systems at the SEPI ESIME Zacatenco IPN, CDMX, Mexico.
His research interests include deregulated markets and optimization in power systems.

David Sebastián Baltazar
Received the Ingeniero Industrial en Eléctrica degree from Instituto Tecnológico de Morelia, México 1990. Received the master and PhD's degree in Electrical Engineering from SEPI, ESIME, IPN, Mexico in 1993 and 1999 respectively.
He is currently professor at the SEPI, ESIME, IPN in Mexico. His research interests include electric systems protection.