Thevenin and Norton discrete equivalent models in electric circuits using companion-circuit analysis

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***Abstract*—This article details a methodology for time-domain (TD) electric circuit analysis based on companion-circuit analysis (CCA). The application of the CCA method is carried out using advanced numerical techniques through the numerical integration of the Trapezoidal Rule (TR). In general, the CCA method allows an representation of electric components through the substitution of Thevenin and Norton equivalent discrete models. These equivalent models allow an efficient and precise solution. The results obtained from the case studies are given in terms of time-steps per period and accuracy. The documented results have been validated with those obtained using the PSCAD/EMTDC® simulator.**

***Keywords*—Companion-circuit, discrete equivalent model, equivalent Norton, equivalent Thevenin, time-step, trapezoidal rule.**

I. Introduction

Currently there are techniques and/or simulators for analyzing electric circuits in the TD with different electric elements, such as resistance (*R*), inductor (*L*) and capacitor (*C*) among others [1]. Digital simulators are tools that only allow obtaining graphical and numerical results, which is convenient in the practical case [2]-[4]. However, it is unlikely that its structure can be internally known. For this reason, in various applications of analysis and study of electric circuits, it is possible to obtain models in a symbolic way and to know in detail their individual or joint representation [1], [5].

The symbolic analysis of electric circuits is established from an interconnection topology through branches and nodes [6]. The network topology allows to represent electric elements by means of a set of variables that represent currents and voltages [6], [7]. The variables can be obtained from the application of energy conservation laws. Also, in the analysis of electric circuits it is convenient to use equivalent models that allow a simplified representation or, where appropriate, that allow the transformation between models, such is the case of the Thevenin and Norton equivalents [8], [9]. Now, an alternative electric circuit analysis is based on CCA detailed in [1], which allows to obtain a representation of electric elements by means of Thevenin and Norton discrete equivalents from the application of a integration method [5], [8], [10].

Fundamentally, this work presents a tool for the analysis of electric circuits in TD base on CCA. The Thevenin and Norton equivalent discrete models is referred to what will be called $\mathrm{CCA_{Th}}$ and $\mathrm{CCA_{N}}$, respectively.

The rest of the work is organized as follows: Section 2 illustrates the equivalent discrete models; Section 3 describes the case studies and their model; Section 4 presents the results obtained graphically and Section 5 summarizes the general conclusions obtained from this experience.

II. Equivalent models for digital components simulation

The using of models and/or equivalent techniques allow the representation of complex electric circuits in a practical and simple way. The equivalent models consist of obtaining a reduced number of equivalent equations of any electric circuit, preserving its main operating characteristics. In this case, there are two equivalent models based on the CCA technique, i.e. the Thevenin and Norton discrete equivalent model [8]. These equivalents can be obtained by applying different numerical integration methods [5], [10]. The Norton discrete equivalent model is the technique most widely used by several EMTP-type simulation platforms, since the analysis technique is given by means of an equivalence of nodal equations [11], [12].

These techniques are suitable for theoretical analysis used to obtain the equivalent equations of different electric components (e.g. inductor, capacitor, among others). The basic principle of the Thevenin and Norton discrete equivalent model through the CCA method, is based on obtaining a set of ordinary algebraic equations (OAEs) to be solved in fixed discrete time-steps (Δt), obtained through the application of Kirchhoff current and voltage laws, i.e. KVL and KCL, respectively.

Initially, it should be considered that the behavior of an electric component, such as inductor or capacitor (see Table I, column 2), can be represented by an ODE written as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{1}$$

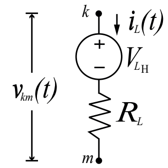
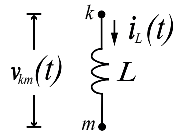
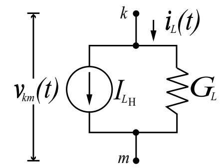
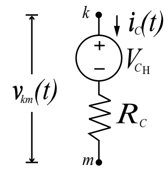
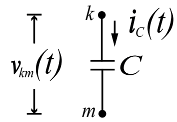
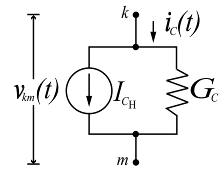
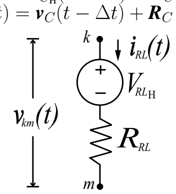
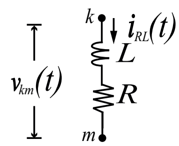
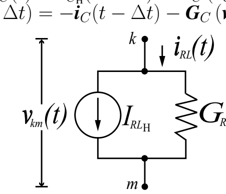
now, integrating (1), with respect to time (t), in the interval t-Δ and t, the solution of the recursive equation is given by,

$$\mathbf{x}_t = \mathbf{x}_{(t-\Delta t)} + \frac{\Delta t}{2}[\mathbf{f}_t + \mathbf{f}_{(t-\Delta t)}] \tag{2}$$

where t, Δt and t-Δt are the present value, the fixed time-step size and the past time value, respectively.

Then, if elements *L* and *C* are considered and applying the TR method in (2), taking as reference a fixed time-step (Δt) [12], the Thevenin and Norton discrete equivalent models are obtained, as shown in Table I (rows 2 and 3).

TABLE I: Thevenin and Norton discrete equivalent models based on CCA technique.

Thevenin discrete equivalent model	**Element**	**Norton discrete equivalent model**
 $R_L = 2L/\Delta t$ $v_L(t) = V_{L_H}(t-\Delta t) + R_L(i_L(t))$ $V_{L_H}(t-\Delta t) = -v_L(t-\Delta t) - R_L(i_L(t-\Delta t))$	 **Inductance (*L*)**	 $G_L = \Delta t/2L$ $i_L(t) = I_{L_H}(t-\Delta t) + G_L(v_L(t))$ $I_{L_H}(t-\Delta t) = i_L(t-\Delta t) + G_L(v_L(t-\Delta t))$
 $R_C = \Delta t/2C$ $v_C(t) = V_{C_H}(t-\Delta t) + R_C(i_C(t))$ $V_{C_H}(t-\Delta t) = v_C(t-\Delta t) + R_C(i_C(t-\Delta t))$	 **Capacitance (*C*)**	 $G_C = 2C/\Delta t$ $i_C(t) = I_{C_H}(t-\Delta t) + G_C(v_C(t))$ $I_{C_H}(t-\Delta t) = -i_C(t-\Delta t) - G_C(v_C(t-\Delta t))$
 $R_{RL} = (2L + R\Delta t)/\Delta t$; $H_{RL} = (1 + (R_L/R))/(1 - (R_L/R))$ $v_{RL}(t) = V_{RL_H}(t-\Delta t) + R_{RL}(i_{km}(t))$ $V_{RL_H}(t-\Delta t) = -H_{RL}(v_{RL}(t-\Delta t)) - R_{RL}(i_{km}(t-\Delta t))$	 **Series Branch (*RL*)**	 $G_{RL} = \Delta t/(2L + R\Delta t)$; $H_{RL} = (1 - RG_L)/(1 + RG_L)$ $i_{RL}(t) = I_{RL_H}(t-\Delta t) + G_{RL}(v_{km}(t))$ $I_{RL_H}(t-\Delta t) = H_{RL}(i_{RL}(t-\Delta t)) + G_{RL}(v_{km}(t-\Delta t))$

Finally, in this reference line, a Thevenin and Norton discrete equivalent models can also be obtained by means of simplified *RL* branches, i.e. the Thevenin and Norton discrete equivalent models are obtained by representing transmission lines of short and medium length is shown in Table I (row 4).

The discret equivalent models are described below

A. Thevenin discrete equivalent model

In general, this type of analysis allows a simplified model to be obtained by means of equivalent equations (OAEs) by applying the CCA method [13]. The result allows to obtain a model that establishes a voltage source in series to an equivalent resistance, as shown in Table I (column 1).

The Thevenin discrete equivalent model is obtained by (2) considering that the elements *L* and *C* are given by

$$v_{km}(t) = V_{\mathrm{H}}(t-\Delta t) + R\left(\boldsymbol{i}_{km}(t)\right) \tag{3}$$

*1) **General solution by applying KVL**:* This type of formulation allows the representation of any electric circuit through a set of OAEs, described by the following expression

$$[\boldsymbol{R}]\boldsymbol{i}(t) = \boldsymbol{v}(t) + \boldsymbol{V}_{\mathrm{H}} \tag{4}$$

where,

$[\boldsymbol{R}]$	:	Equivalent resistance matrix.
$\boldsymbol{i}(t)$	:	Vector of branch currents.
$\boldsymbol{v}(t)$	:	Vector of external voltage sources.
$\boldsymbol{V}_{\mathrm{H}}$	:	Vector voltage sources of past history terms.

B. Norton discrete equivalent model

This model makes it possible to obtain an equivalent, which unlike the Thevenin discrete equivalent model, allows to obtain an Norton discrete equivalent model formed by a current source and resistance connected in parallel (see Table I. column 3) [14].

Now, from (2) and considering elements *L* and *C*, we obtain

$$i_{km}(t) = \boldsymbol{I}_{\mathrm{H}}(t-\Delta t) + G\left(v_{km}(t)\right) \tag{5}$$

*1) **General solution using KCL**:* It is possible to obtain a discrete Norton equivalent model described as follows,

$$[\boldsymbol{G}]\boldsymbol{v}(t) = \boldsymbol{i}(t) + \boldsymbol{I}_{\mathrm{H}} \tag{6}$$

where,

$[\boldsymbol{G}]$	:	Equivalent conductance matrix.
$\boldsymbol{v}(t)$	:	Vector of nodal voltage.
$\boldsymbol{i}(t)$	:	Vector of external current sources.
$\boldsymbol{I}_{\mathrm{H}}$	:	Vector current sources (past history terms).

The solution process of the Thevenin and Norton discrete equivalent models is given by means of a LU decomposition process of the equivalents resistance and conductance matrices, $[\boldsymbol{R}]$ and $[\boldsymbol{G}]$, respectively, this in order to avoid matriz inverse calculation.

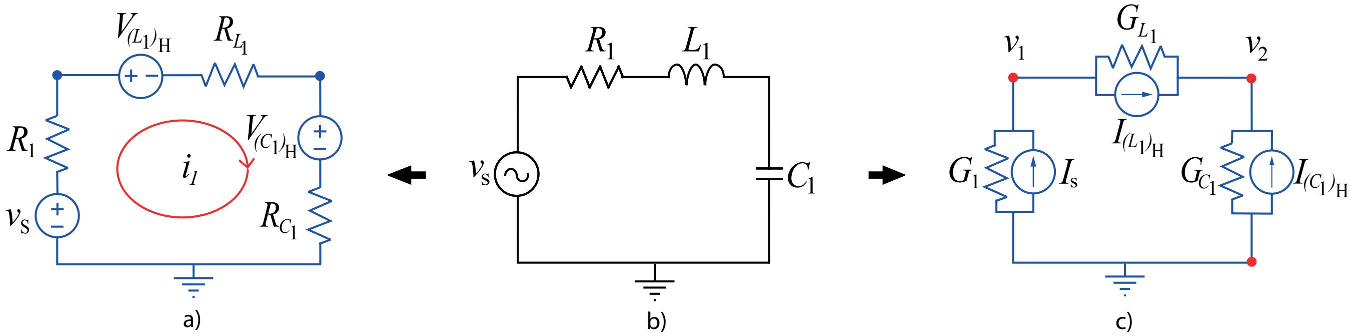


Fig. 1: Series *RLC* electric circuit: a) Equivalent circuit by using CCA_{Th}, b) Electric circuit, c) Equivalent circuit by using CCA_{N}.

III. Representation of equivalent models

The illustrated TD methodology is now applied by substituting Thevenin and Norton discrete equivalent models in different case studies as shown in Figs. 1 and 2, to obtain the solution.

The CCA-based electric circuit analysis technique takes into consideration the following points:

- The analysis is considered by applying the Thevenin and Norton discrete equivalent models (CCA_{Th} and CCA_{N}, respectively), which using TR as a numerical integration method.
- The CCA_{Th} and CCA_{N} methods are applied to the case studies to be presented.
- Assessment of currents and voltages (state variables) is given by the CCA_{Th} and CCA_{N} methods, respectively.
- The results obtained from the case studies detail a comparison between the CCA_{Th} and CCA_{N} methods in terms of time-steps per period and accuracy.
- The validation is verified by direct comparison against the PSCAD/EMTDC® simulator.

A. Series RLC electric circuit

Fig. 1a shows an electric circuit with a sinusoidal voltage source connected in series with *R*, *L* and *C* elements. This test circuit shows a simple analysis application using the CCA method, where the determination of state variables is considerably reduced. In particular, the Thevenin and Norton discrete equivalent models consider the state variables of currents and voltages, respectively.

From the electric circuit of Fig. 1b, the application of the Thevenin discrete equivalent model becomes the equivalent model presentation of Fig. 1a, where a current direction is assumed and the KVL is applied to obtain the equivalent model represented by the following equation,

$$\left[R_1+R_{L_1}+R_{C_1}\right]\times\left[\boldsymbol{i}_1\right]=\left[\boldsymbol{V}_s\text{-}\boldsymbol{V}_{(L_1)_{\mathrm{H}}}\text{-}\boldsymbol{V}_{(C_1)_{\mathrm{H}}}\right] \tag{7}$$

where $\boldsymbol{i}_1$ is the state variable obtained by applying the Thevenin discrete equivalent model.

Now, if we consider the application of the Norton discrete equivalent model from Fig. 1b, the model becomes the equivalent circuit of Fig. 1c. This type of model allows an analysis by means of nodal incidence applying the KCL.

Therefore, the model obtained of Fig. 1c is given by the following matrix relationship,

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_{L_1}} & -\frac{1}{R_{L_1}} \\ -\frac{1}{R_{L_1}} & \frac{1}{R_{L_1}}+\frac{1}{R_{C_1}} \end{bmatrix}\times\begin{bmatrix}\boldsymbol{v}_1\\ \boldsymbol{v}_2\end{bmatrix}=\begin{bmatrix}\boldsymbol{I}_s\text{-}\boldsymbol{I}_{(L_1)_{\mathrm{H}}}\\ \boldsymbol{I}_{(L_1)_{\mathrm{H}}}\text{-}\boldsymbol{I}_{(C_1)_{\mathrm{H}}}\end{bmatrix} \tag{8}$$

where the conductance matrix is symmetric and of order 2x2.

Note that, the Thevenin equivalent discrete model is only represented by one state variable (current), while the Norton equivalent discrete model by two state variables (voltages).

B. Electric network with series RL branches

The electrical network shown in Fig. 2b consists of a sinusoidal voltage source, three busbars, a short transmission line represented by *RL* branch, a medium length transmission

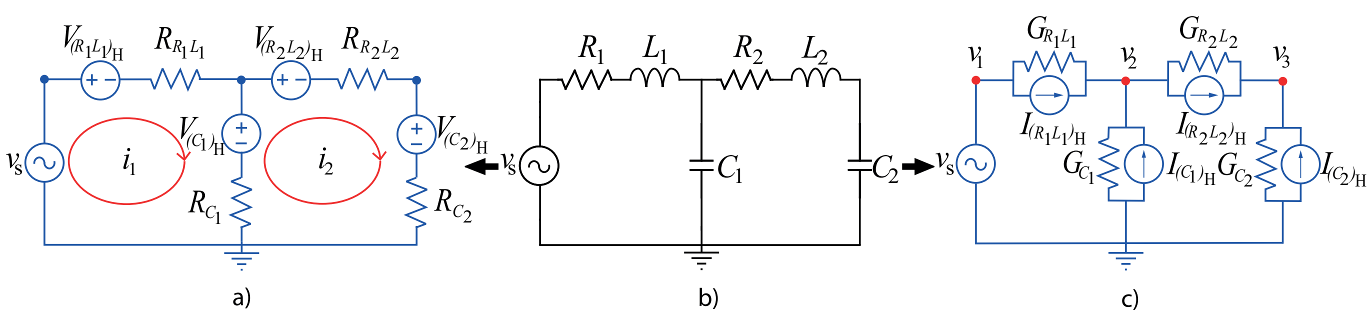


Fig. 2: Electric network with *RL* branches: a) Equivalent diagram by applying CCA_{Th}, b) Electric network, c) Equivalent diagram by applying CCA_{N}.

line represented by a nominal PI model. The representation of the Thevenin and Norton equivalent discrete model is given from the electrical network of Figures 2a and 2c, respectively.

From the electrical network in Fig. 2a, the Thevenin equivalent discrete model is obtained, which corresponds to two state variables, i.e. mesh currents are considered. This equivalent model consists of simplified RL branches, as detailed in Table I, (row 4). Therefore, the Thevenin equivalent discrete model is obtained from the application of the KVL in the electric network of Fig. 2c, which is given by

$$\begin{bmatrix} R_1+R_{L_1}+R_{C_1} & \text{-}R_{C_1} \\ \text{-}R_{C_1} & R_{C_1}+R_2+R_{L_2} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{i}_1 \\ \boldsymbol{i}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_s\text{-}\boldsymbol{V}_{(R_1L_1)_{\mathrm{H}}}\text{-}\boldsymbol{V}_{(C_1)_{\mathrm{H}}} \\ \boldsymbol{V}_{(C_1)_{\mathrm{H}}}\text{-}\boldsymbol{V}_{(R_2L_2)_{\mathrm{H}}}\text{-}\boldsymbol{V}_{(C_2)_{\mathrm{H}}} \end{bmatrix} \tag{9}$$

In this case, the equivalent model of (9) consists of an equivalent resistances matrix with dimensions 2x2.

For the application of the Norton equivalent discrete model to the electric network, the circuit shown in Fig. 2c results. Now if we apply the LCK, we obtain the Norton equivalent discrete model given by (10), which is composed of a 3x3 equivalent conductance matrix. This model consists of three state variables (i.e. three nodal voltages).

$$\begin{bmatrix} \frac{1}{R_{R_1L_1}} & \frac{1}{R_{R_1L_1}} & 0 \\ \frac{1}{R_{R_1L_1}} & \frac{1}{R_{R_1L_1}}+\frac{1}{R_{R_2L_2}}+\frac{1}{R_{C_1}} & \frac{1}{R_{R_2L_2}} \\ 0 & \frac{1}{R_{R_2L_2}} & \frac{1}{R_{R_2L_2}}+\frac{1}{R_{C_2}} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{v}_1 \\ \boldsymbol{v}_2 \\ \boldsymbol{v}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_s\text{-}\boldsymbol{I}_{(R_1L_1)_{\mathrm{H}}} \\ \boldsymbol{I}_{(R_1L_1)_{\mathrm{H}}}\text{-}\boldsymbol{I}_{(R_2L_2)_{\mathrm{H}}}\text{-}\boldsymbol{I}_{(C_1)_{\mathrm{H}}} \\ \text{-}\boldsymbol{I}_{(R_2L_2)_{\mathrm{H}}}\text{-}\boldsymbol{I}_{(C_2)_{\mathrm{H}}} \end{bmatrix} \tag{10}$$

IV. Comparison of Equivalent Models Performance

The Thevenin and Norton equivalent discrete models ($\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$, respectively) above described in the TD (using the TR method), are now applied for the analysis and solution of the electric circuits that are presented in the case studies, according to Figs. 2b and 2b. The parameters are given in per unit (p.u.) in Appendix A. In all cases the solution is determined using 512 time-steps per period. The results obtained in each case study are compared in terms of time-steps per period and accuracy, taking as reference those obtained using PSCAD/EMTDC®.

The equivalent models obtained were implemented and encoded using the MATLAB® 2018b (64-bit) emulator, installed on a DELL Inspiron portable computer with a corei7 processor, clocked at 2.60 GHz.

A. Case study 1: Series RLC circuit

In this example, the Thevenin and Norton equivalent discrete models ($\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$, respectively) are used for the analysis of the *RLC* electric circuit of Fig. 1b. The $\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$ methods establish as state variables mesh currents and nodal voltages, respectively. Then, the obtained models consist of one and two state variables, respectively. The solution process is given by the same operating conditions, i.e. parameters and integration step. The simulation time for the equivalent models and PSCAD/EMTDC® with an integration step of 3.25 μs.

Please observe the close agreement obtained between the responses obtained by applying the $\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$ methods with respect to the response obtained with PSCAD/EMTDC®, as shown in Fig. 3. Again, a time period of solution between 0.2461 and 0.2628 sec. shows a close coincidence of the zoomed response from 0.25022 to 0.2503 sec. in Fig. 3.

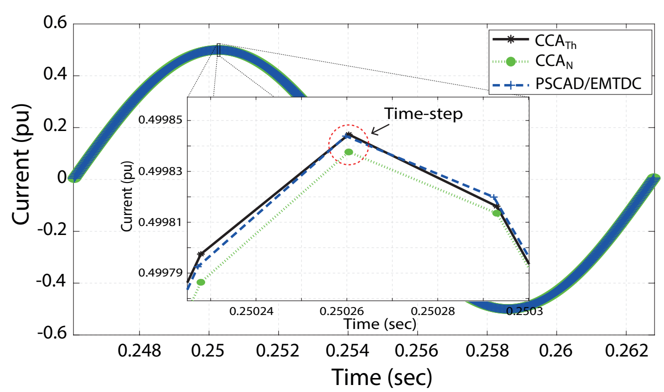


Fig. 3: Waveform of the current and close-up of I_{L_1}.

Again, please observe zoomed response of Fig. 3, where the location of the established time-steps for the $\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$ methods, including that of PSCAD/EMTDC®. The $\mathrm{CCA_{Th}}$ and $\mathrm{CCA_N}$ methods show a close agreement with respect to the response obtained by PSCAD/EMTDC®. However, the $\mathrm{CCA_{Th}}$ method shows better agreement with PSCAD/EMTDC®.

For the voltage waveform (V_{C_1}) in Fig. 4 shows a solution time period between 0.25 and 0.2664 sec., which shows close agreement, as shown by the zoomed response from 0.25438 to 0.2544 sec. In this case, the response obtained by applying the CCA_{Th} method presents a good agreement with respect to PSCAD/EMTDC®, but the CCA_N method shows a closer response than the obtained by the CCA_{Th} method.

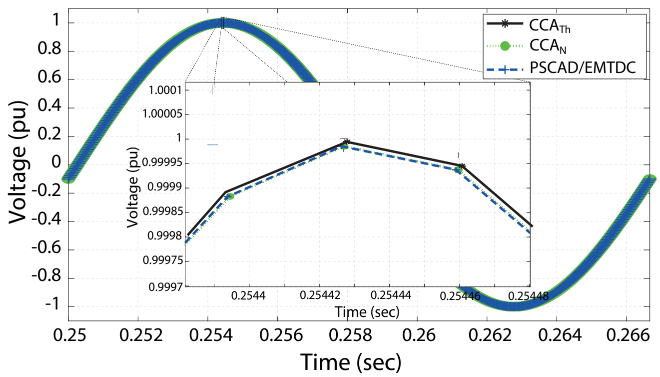


Fig. 4: Waveform of the voltage and close-up of V_{C_1}.

Note that the calculation of the relative error between the current and voltage waveforms is extremely small between the methods (CCA_{Th} and CCA_N) and PSCAD/EMTDC®.

B. Case study 2: Electric network with series RL branches

The solution is obtained for the electric network of Fig. 2b. The equivalent models obtained are represented by two and three state variables obtained using the application of the CCA_{Th} and CCA_N methods (mesh currents and nodal voltages, respectively). The waveforms obtained by the equivalent CCA_{Th} and CCA_N methods (using the TR method) are compared against those obtained by the PSCAD/EMTDC® simulator.

For the $\boldsymbol{I}_{(R_2L_2)}$ current response presented in Fig. 5, the time period is between 0.2477 and 0.2644 sec. In particular, please observe the zoomed response from 0.2517 to 0.2523 sec., which shows the comparison between the solutions obtained from the CCA_{Th} and CCA_N methods, including PSCAD/EMTDC®. The relative error between the CCA_{Th} method solution with respect to PSCAD/EMTDC® is about 0.42%, while the CCA_N method solution with respect to PSCAD/EMTDC® has a relative error around 2.36%.

Fig. 6 shows the solution comparison of the voltage waveforms V_{C_2}, which shows a time period between 0.2518 and

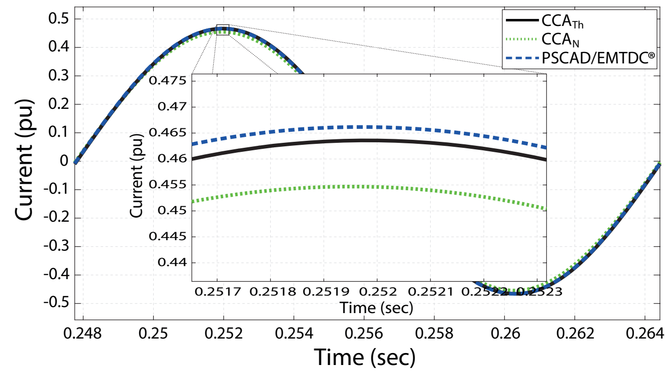


Fig. 5: Current waveform $\boldsymbol{I}_{(R_2L_2)}$.

0.2685 sec. Please observe zoomed region of 0.25525 and 0.25568 sec., a close agreement has been obtained between the responses of the CCA_{Th} and CCA_N methods compared against the PSCAD/EMTDC®. The maximum relative error obtained between the CCA_{Th} and CCA_N methods with respect to PSCAD/EMTDC® is around 0.17% and 0.52%, respectively.

The maximum relative error between both responses is below 1%, which satisfactorily validates the CCA_{Th} and CCA_N methods.

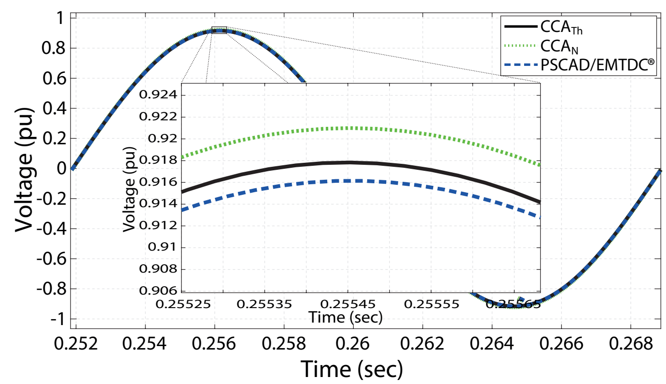


Fig. 6: Voltage waveform V_{C_2}.

Finally, based on the results obtained from the case studies 1 and 2 by applying the CCA_{Th} and CCA_N methods, the following features of each particular method are summarized in Table II.

TABLE II: Features of CCA_{Th} and CCA_N equivalent methods.

Method	**Features**
CCA_{Th}	Efficient for the analysis of small-scale electric circuits. Very precise small-scale electric circuits. The formulation allows in a simple way the representation of state variables in small-scale electric circuits.
CCA_N	Efficient for the analysis of small, medium and large scale electric circuits. The application is simple and accurate. Allow an efficient representation of the state variables of small, medium and large-scale electric networks.

V. Conclusions

The importance of this research work is its educational application, which presents two equivalent alternatives based on the CCA, which is used for the analysis of electric circuits. However, it can be applied to a wide range of case studies. In general, the CCA-based Thevenin and Norton equivalent discrete models are of direct and simple application in small-scale electric circuits.

Also, its simplicity and accurate have been demonstrated through the case studies presented. However, it is observed that for the Thevenin discrete equivalent model (CCA_{Th}), its application is limited to the analysis of small-scale electric circuits. Its application to the solution of larger-scale electric circuits may turn to be difficult and/or prohibitive. On the contrary, regarding the Norton discrete equivalent model (CCA_N); its formulation and analysis through nodal incidence, which allows its practical application in medium and large-scale electrical circuits.

Now, in terms of accuracy, from case study 2 by comparing the voltage and current waveforms, which obtained by applying the CCA_{Th} and CCA_N methods, with respect to the responses of the PSCAD/EMTDC® simulator. In particular, the maximum relative error between the CCA_{Th} and CCA_N methods, with respect to PSCAD/EMTDC® is approximately 0.29% and 1.44%, respectively.

Therefore, the CCA_{Th} and CCA_N methods, on average, turn out to be efficient and accurate electric circuit analysis tools. The solution can be obtained efficiently while maintaining accuracy.

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VI. Appendix

A. Parameters

TABLE III: Case study 1 (p.u.)

$V_m =$	$sin(wt)$	$L_1 =$	0.01
$R_1 =$	0.2	$C_2 =$	0.5

TABLE IV: Case study 2 (p.u.)

$V_m =$	$sin(wt)$	$R_1 =$	0.5	$L_1 =$	0.1	$C_1 =$	0.5
		$R_2 =$	0.5	$L_2 =$	0.1	$C_2 =$	0.5

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