Analysis of a Compensation Fiber with Dispersion Curves Inverse to a Standard Fiber

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***Abstract*—We perform a dispersion analysis for a single mode standard fiber to propose a multichannel compensating fiber with similar physical characteristics that allow their splicing.**

We start with a review of dispersion compensation fibers with known refractive index profiles to, by tailoring them, obtain a dispersion curve with a magnitude inverse in sign to that of the standard fiber.

Dispersion analysis is performed using the Spectral Parameter Power Series (SPPS) numeric method, which makes evaluating multiple profiles (which were used to obtain the desired dispersion behavior) easy and practical, in relatively short amounts of time.

Keywords—Dispersion Reverse Fiber, Dispersion compensation, Wavelength Division Multiplexing, Spectral Parameter Power Series, Single Mode Fiber.

I. Introduction

In fiber optics communications the third optical window (1550 nm) is one of the most used ones due to its high capacity and low loss. However, the dispersion phenomenon limits the capacity and range in a transmission system. Dispersion refers to the spectral delay (deformation) of an optical pulse as it propagates along the fiber [1]. Nowadays, some DCFs (Dispersion Compensating Fibers) can be found in the market, but they are limited to compensating only one wavelength, being impractical incorporating them to multichannel systems [2]. Reverse Dispersion Fiber (RDF) can also be found; they are ideal for multichannel systems but only in narrow wavelength ranges [3]. In this paper we are looking for a way to compensate the chromatic dispersion for a standard Single Mode Fiber (SMF) in a wide range of wavelengths, that is, for possible applications to multichannel WDM systems. This would be accomplished by placing a section of the proposed fiber after a section of a SMF to compensate the dispersion in each wavelength of the considered spectrum width, allowing the transmission conditions for the multichannel system to improve.

Changes in the optical fiber's refractive index profile allow the manipulation of the transmission characteristics, such as propagation modes, their group velocity, propagation constants and chromatic dispersions [4].

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To accomplish this proposal, the concept of dispersion and its components, and the means to calculate it will be introduced in Section II. State of the art fibers will also be presented, with which performance comparisons can be made. Section III will present the process that allows the determination of the mathematical model that should be solved to determine the guided modes and their dispersion. The basic aspects of the proposed SPPS method with which the required solutions can be found will also be presented. Section IV describes the performed tests and the obtained results, and finally we present the obtained conclusions.

II. Dispersion in Optical Fibers

A. Dispersion

Dispersion is the phenomenon of the splitting of waves of different frequency as they go through a material. It is also the name given to the property of velocity variation with the wavelength. All materials are dispersive (some of them more than others), and dispersion affects all waves.

Its effect is, that when a pulse is transmitted through the optical fiber, it does not maintain its shape, but widens in time as it propagates.

Dispersion increases due to:

- The length of the optical fiber of the link.
- The bit rate (pulse frequency) of the system.

Chromatic dispersion D_C is composed by two types of dispersion [1]:

- The material dispersion D_M, and
- The waveguide dispersion D_W.

That is

$$D_C = D_M + D_W. \tag{1}$$

Material dispersion occurs because the different wavelengths travelling through a material (in this case glass or plastic) travel with different velocities. That is, the refractive index n of the material (and therefore the speed of light) varies with the wavelength. This dispersion is quite significative in SMFs, but for Multimode Fibers (MMF) it is negligible because in these the intermodal dispersion is more significative.

Material dispersion is given by [1]

$$D_M = -\frac{2\pi}{\lambda^2}\left(\frac{dn_{2g}}{d\omega}\right) = \frac{1}{c}\left(\frac{dn_{2g}}{d\lambda}\right) \quad (2)$$

where n_{2g} is the group refractive index, c is the speed of light in a vacuum, λ is the wavelength and ω is the frequency.

The waveguide dispersion D_W is a complex effect and is mainly caused by the shape of the refractive index profile of the fiber's core, which has a significant effect on the group velocity. This is because the electric and magnetic fields that are part of a light pulse extend beyond the core. This amount that the fields share between the cladding and the core has a strong dependence on the wavelength. At larger wavelengths, the amount of the electromagnetic wave that extends over the cladding grows, and it is calculated as [1]

$$D_W = -\frac{n_2\Delta}{c\lambda}\left(V\frac{d^2(Vb)}{dV^2}\right) \quad (3)$$

where V is the normalized frequency $V = 2\pi a(n_1^2 - n_2^2)^{1/2}/\lambda$, n_1 and n_2 are the refractive indexes of the core and cladding, a is the radius of the fiber's core, b is the normalized propagation constant $b = \beta^2(k_0^2(n_1 - n_2))$, β is the phase (or propagation in absence of attenuation) coefficient, k_0 is the wavenumber in the free space and Δ is the relative difference of refractive indexes

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}.$$

Sometimes the two components of the chromatic dispersion act in opposite direction: the material dispersion takes on negative values, while the waveguide dispersion has positive and negative values. Therefore, in any fiber there is a specific wavelength in which both effects are neutralized, and the chromatic dispersion is null (or negligible). An alternative way to calculate it is [1]

$$D_C = -\frac{2\pi c}{\lambda^2}\left(2\frac{d\tilde{n}}{d\omega} + \frac{d^2\tilde{n}}{d\omega^2}\right) \quad (4)$$

where $\tilde{n}$ is the modal index at the operating wavelength.

B. Reverse Dispersion Fiber (RDF)

RDFs are fibers that have both the dispersion coefficient and its slope opposite to those of a standard SMF and are known as IDF (Inverse Dispersion Fiber) or RDF [3]. The transmission designs where these fibers are used consist of a SMF fiber with positive slope and dispersion followed by an RDF or an IDF fiber with negative slope and dispersion around the third window (1550 nm). These composed transmission lines are also called Dispersion Management Lines (DML) or Dispersion Management Fibers (DMF) [3]. This kind of fibers are used to achieve a net flat dispersion, desirable for WDM transmissions.

In 1999, the research team of The Furukawa Electric CO company designed and built an RDF fiber that could compensate not only the dispersion of a wavelength, but also the dispersion slope. The dispersion obtained in the RDF is completely inverse to that of a standard SMF (without reaching a zero dispersion after the compensation to avoid non-linear effects). However, they achieved this only in a wavelength range of 1530 to 1570 nm (see Fig 1). The developed fiber had good optical and mechanical properties, and its viability for practical applications was confirmed, proving that it was an optimal fiber for DWDM systems [5].

SMF+IDF was widely accepted as a next-gen WDM transmission line, because IDF induced a lot of advantages like low losses [3]. The SMF+RDF combination seeks to provide said flatness in the largest wavelength range possible. Therefore, obtaining a broadband dispersion flatness is highly desirable.

III. Analysis of Guided Modes in Optical Fibers of Graded Index

Mathematically, we solve the basic equation for the wave propagation in graded index fibers [6]

$$\frac{d^2}{dr^2}\Psi + \frac{1}{r}\frac{d}{dr}\Psi + \left[k_0^2 n^2(r) - \beta^2 - \frac{m^2}{r^2}\right]\Psi = 0, \quad r \in (0, a] \quad (5)$$

where Ψ stands in this case for the H_z component of the magnetic field, $n(r)$ is the refractive index profile depending on the radius and m is a mode parameter given by [6]

$$m = \begin{cases} 1 & TE \text{ and } TM \text{ modes} \\ l+1 & EH \text{ modes } (l \geq 1) \\ l-1 & \text{HE modes } (l \geq 1). \end{cases}$$

The integer l is the mode order in the azimuthal direction θ.

In graded index fibers, the refractive index varies in the core while it is constant in the cladding. Thus, the equation is usually solved for the core and cladding separately, and the solutions and their derivatives are equated in the boundary. The solutions are denoted by Ψ_{core} in the core and Ψ_{clad} in the cladding.

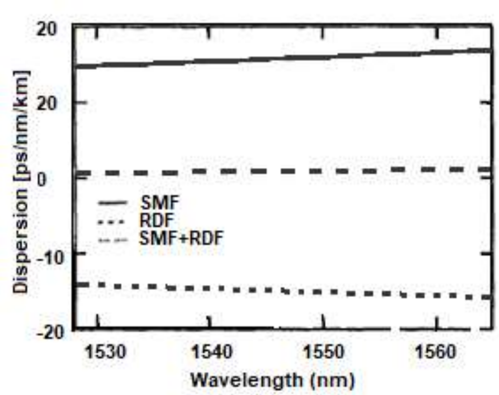

Fig. 1 Total dispersion of the RDF in [5].

Using the change of variable $U(r) = r^{-1/2}\Psi$, (5) can be written as [7]

$$-U''(r)\left(\beta^2 + \frac{m^2 - \frac{1}{4}}{r^2} - k_0^2 n^2(r)\right)U(r) = 0. \qquad (6)$$

Making a new change of variable $x = r/a$ the function $u(x) = U(ar)$ is obtained and (6) is written as

$$-u''(x) + \left(a^2\beta^2 + \frac{m^2 - \frac{1}{4}}{x^2} - a^2 k_0^2 n^2(ax)\right)u(x) = 0. \qquad (7)$$

For this new function and under the normalization of the radius of the core, the boundary conditions are

$$u_{core}(1) = u_{clad}(1), \quad \frac{du_{core}}{dx} = \frac{du_{clad}}{dx} \qquad (8)$$

The solution for the cladding, this being homogeneous, is known as [6]

$$u_{clad}(x) = C\sqrt{x}K_m(\gamma x) \qquad (9)$$

where K_m is the modified second type Bessel function, $\gamma := a(\beta^2 - k_0^2 n_2^2)^{1/2}$ and C is an arbitrary constant. The characteristic equation is [7]

$$2K_m(\gamma)u'(1) - \big((1+2m)K_m(\gamma) -$$

$$2\gamma K_{m+1}(\gamma)\big)u(1) = 0. \qquad (10)$$

The guided modes correspond to those β and k_0 pairing values that satisfy the characteristic equation along with the condition for wave propagation

$$k_0^2 n_2^2 < \beta^2 < k_0^2 n_1^2 \,. \qquad (11)$$

It must be remembered that for a graded index fiber we consider

$$n_1 = \max_{0 \le x \le 1} n^2(ax).$$

The so called "α" or "power" profiles are usually considered, which are defined in the core by [8]

$$n(r) = n_1\left(1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}\right)^{1/2}, \; 0 \le r \le a. \qquad (12)$$

The most common graded fibers use a quasi-parabolic profile ($\alpha \approx 2$), while a triangular profile is generated with $\alpha = 1$. Here they will be used for multiple layers, for which (12) must be shifted, its width adjusted, its maximum value n_1 adequate in amplitude and tested with different values for α, all of this to generate the required dispersion compensation.

A. Application on the SPPS method for the obtention of guided modes.

The recently developed SPPS method [9, 10] offers representations for the solutions of Sturm-Liouville equations in form of power series. Some of its most notable applications can be consulted in [11]. The main equation to analyze here belongs to the said type of equations and has the form

$$-u''(x) + \left(\frac{m^2 - \frac{1}{4}}{x^2} + q(x)\right)u(x) = \lambda r(x)u(x), \qquad (13)$$

where $q(x)$ and $r(x)$ are given continuous complex functions and λ is the so-called spectral parameter. More specifically, this belongs to the perturbed Bessel equations, and has been studied in different publications (see, e.g. [12]).

The application of the SPPS method for the obtention of a general solution of (13) requires a particular solution v that does not become zero in the range (0, 1] of the homogeneous equation

$$-v''(x) + \left(\frac{m^2 - \frac{1}{4}}{x^2} + q(x)\right)v(x) = 0. \qquad (14)$$

If $q(x) \ge 0$, $x \in (0, 1]$, then according to [7] v y v' have the series representation

$$v(x) = x^{m+1/2}\sum_{k=0}^{\infty} \tilde{Y}^{(2k)}(x) \qquad (15)$$

and

$$v'(x) = \left(m + \frac{1}{2}\right)x^{m-1/2}\sum_{k=0}^{\infty} \tilde{Y}^{(2k)} +$$

$$x^{-(m+1/2)}\sum_{k=1}^{\infty} \tilde{Y}^{(2k-1)} \qquad (16)$$

where $Y^{(j)}$ is defined for $j = 0, 1, 2, \ldots$ as

$$\tilde{Y}^{(0)}(x) \equiv 1,$$

$$\tilde{Y}^{(j)}(x) = \begin{cases} \int_0^x \tilde{Y}^{(j-1)}(t)t^{2m+1}q(t)dt, & j \text{ odd} \\ \int_0^x \tilde{Y}^{(j-1)}(t)t^{-(2m+1)}dt, & j \text{ even.} \end{cases}$$

In the next step of the SPPS method, the solution v is used to build the unique (up to a multiplicative constant) bounded solution of the equation for any value of λ. This solution has the form [7]

$$u(x) = v(x)\sum_{n=0}^{\infty}\lambda^n \tilde{X}^{(2n)}(x) \tag{17}$$

where the recursive relations for $n = 0, 1, 2, \ldots$ are defined by

$$\tilde{X}^{(0)} \equiv 1,$$

$$\tilde{X}^{(n)}(x) = \begin{cases} \int_0^x v^2(t)r(t)\tilde{X}^{(n-1)}(t)dt, & n \text{ impar} \\ -\int_0^x \frac{\tilde{X}^{(n-1)}(t)dt}{v^2(t)}, & n \text{ par.} \end{cases} \tag{18}$$

As it occurs with Taylor series, where the solutions approach more precisely in the vicinity of a central point, the series (17) is more precise near the origin, and its precision deteriorates as λ increases. If necessary, a process called spectral shift can be used to generate precise solutions around an arbitrary parameter $\lambda = \lambda_0$ [7]. For this, (13) is rewritten in the form

$$-u''(x) + \left(\frac{m^2 - \frac{1}{4}}{x^2} + q(x) - \lambda_0 r(x)\right)u(x) =$$

$$(\lambda - \lambda_0)\, r(x)\, u(x). \tag{19}$$

For a fixed β, (7) can be written as

$$-u''(x) + \left(\frac{m^2 - \frac{1}{4}}{x^2} + a^2\beta^2\right)u(x) = a^2 k_0^2 n^2(ax)u(x) \tag{20}$$

from where $\lambda := a^2 k_0^2$ and for $\lambda = 0$, $v(x) = c\sqrt{x}I_m$□β□ is a particular solution.

Applying the SPPS method we can see that the value $\alpha\beta$ is too big and the particular solution v grows rapidly. The values of the parameter k_0 must satisfy (11), so it is necessary to apply the spectral shift.

Equation (7) can be written as:

$$-u''(x) + \left(\frac{m^2 - \frac{1}{4}}{x^2} + a^2\left(\beta^2 - k_1^2 n^2(ax)\right)\right)u(x) =$$

$$a^2\left(k_0^2 - k_1^2\right)n^2(ax)u(x) \tag{21}$$

where $k_1 = \beta/n_1$. From (21) we have $q(x) := a^2(\beta^2 - k_1^2 n^2(ax))$, $r(x) := n^2(ax)$ and $\lambda = a^2\left(k_0^2 - k_1^2\right)$ with regards to (19). The solution v from (13), built as previously explained, does not have other zeros in $[0, a]$ except at $x = 0$.

The characteristic equation (10) can then be written in terms of formal powers as [7]

$$2K_m(\gamma)\left(v(1)\sum_{n=0}^{\infty}\lambda^n\tilde{X}^{(2n)}(1) - \frac{1}{v(1)}\sum_{n=1}^{\infty}\lambda^n\tilde{X}^{(2n-1)}(1)\right)$$

$$-\left((1+2m)K_m(\gamma) - 2\gamma K_{m+1}(\gamma)\right)\cdot$$

$$\left(v(1)\sum_{n=0}^{\infty}\lambda^n\tilde{X}^{(2n)}(1)\right) = 0. \tag{22}$$

The solutions of this equation allow to find the propagated modes in the fiber. Through the controlled manipulation of the profile the desired dispersion curve will be reached.

IV. TEST AND RESULTS

As previously mentioned, by modifying the refractive index profile the chromatic dispersion can be modified. The parameters to be modified in the different layers of the core are:

- Refractive indexes
- Radii
- Relative differences of refractive index (Δ)
- Powers for the power profile.

The SPPS method for obtaining solutions of (21) that meet the characteristic equation (22) was implemented via code using Matlab 2019a in a laptop with an Intel core i5-3320M at 2.6 GHz processor and with a RAM of 4 GB, and a 64-bit operating system. Average run times of 60 s where obtained.

To design a refractive index profile that allowed to obtain a chromatic dispersion inverse to that of a standard SMF, we started with a profile available in the literature [4] (see Table I Reference Profile RF and Fig. 2), with both negative chromatic dispersion and slope behavior.

At the start of the analysis the first drawback was visualized. As the initial profile had a very large core, the needed radius had to be reduced (4.5 μm to be able to splice with a standard SMF) This was achieved by reducing the radius with very little variations, and simultaneously modifying the remaining parameters so that the behavior of the chromatic dispersion was maintained.

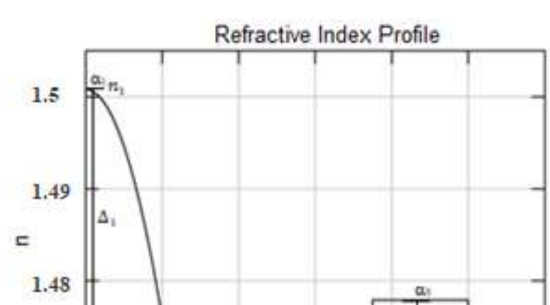

TABLE I. CHARACTERISTICS OF THE PROPOSED REFRACTIVE INDEX PROFILES.

	Refractive Index		Radii (µm)			Relative difference of refractive index (%)			*α*		
	n_1	n_2	*a*	*b*	*c*	Δ_1	Δ_2	Δ_3	α_1	α_2	α_3
RP	1.50	1.472	1.8	5.7	7.6	1.9	–0.04	0.04	2	-	-
1	1.49	1.46	2.1	3.3	4.5	2.2	–2	0.185	2.6	-	-
2	1.52	1.49	2	3.4	4.5	2	–2.5	0.95	3	1.5	1
3	1.52	1.496	2	3.4	4.5	2	–2.5	0.95	3	1.5	1

Having determined the effects caused by modifying the parameters like the refractive indexes, radii, relative differences between refractive indexes and α, simultaneous modifications were made seeking to obtain the inverse dispersion curve. This is how the three proposed designs were obtained.

The second and third proposed designs are obtained from the refractive index profile where α can be modified in each layer. Starting from the known profile shown in Table 1, the Proposed Design 1 was the best design obtained performing as shown in Fig. 3. Being able to modify the powers α_i, we can modify the refractive index profile with better precision, considering that when modifying α_2 and α_3 they have the same behavior that α_1 has in the central layer.

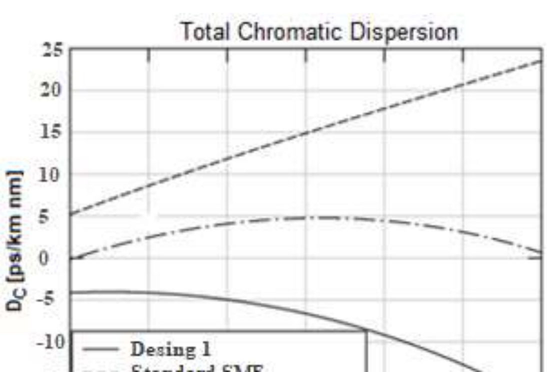

TABLE II. TOTAL DISPERSION FLATNESS OF THE PROPOSED DESIGNS IN WIDE RANGES.

Wavelength (nm)	Proposed 1 (ps/ nm·km)	Proposed 2 (ps/nm·km)	Proposed 3 (ps/nm·km)
1400-1700	4.782	9.33	10.116
1400-1632	4.782	3.386	3.906
1400-1600	4.782	3.386	2.199

TABLE III. TOTAL DISPERSION FLATNESS OF THE PROPOSED DESIGNS FOR EACH OPTICAL BAND

Band	Range (µm)	Proposed 1 (ps/nm·km)	Proposed 2 (ps/nm·km)	Proposed 3 (ps/nm·km)
E	1.4-1.46	2.775	2.226	**1.854**
S	1.46-1.53	1.837	0.840	**0.087**
C	1.53-1565	**0.170**	0.469	0.830
L	1.565-1.625	**0.830**	2.252	2.819
U	1.625-1.675	**1.942**	2.026	3.721

In Table II the flatness of the three proposed designs is analyzed over a wide range of wavelengths, where the Proposed Design 1 (see Fig. 3) has a better flatness. This is obtained as the subtraction of the highest value of the chromatic dispersion minus the lowest value in the range considered.

For the Proposed Designs 1-3 we have that for each kilometer of standard fiber we require 764 m, 931 m, and 688 m of reverse curve fiber, respectively to compensate the dispersion.

As can be seen, in a wavelength range so wide it is difficult to maintain the flatness, since some proposed designs maintain a curvature that keeps a notable difference between the lowest and the highest dispersion. Therefore, we analyze each transmission band separately (E, S, C, L, and U) to see which design achieves a better compensation. The analysis of the E band will be partial, since this band goes from 1360 to 1460 nm and the designs were analyzed starting at 1400 nm. This is because there exists a calculation error in the code around 1375 nm with the proposed profile.

In Table III the total flatness analysis is shown for each band, highlighting those where the best dispersion flatness was obtained. In Fig. 4, Fig. 5, and Fig. 6 the best results of the S, C, and L bands are shown, respectively, where the dispersion flatness is lower to 1 ps/nm·km.

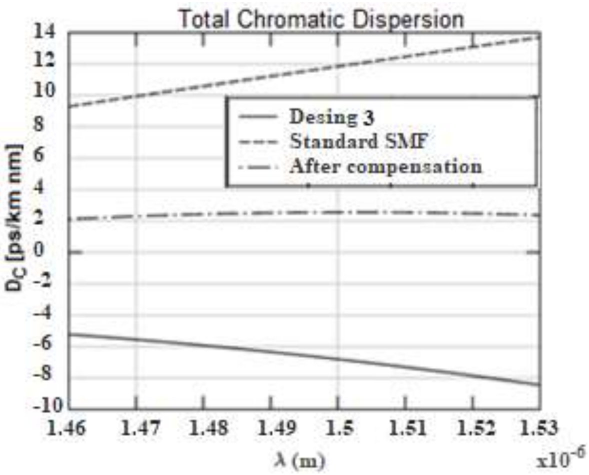

Fig. 4 Total Dispersion in the S band.

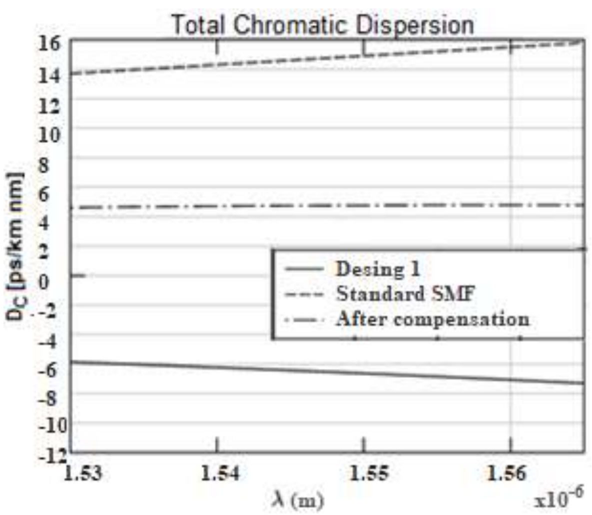

Fig. 5 Total Dispersion in the C band.

In the C band it can be observed (see Fig. 5) that in the three designs the dispersion maintains almost constant, the first design being the one with a better flatness with a difference of 0.17 ps/nm·km; a flatness so small in this band represents a big advantage for third window transmissions.

In the E and U bands, the flatness difference is larger, being that the best designs have a flatness of around 2 ps/nm·km.

The performance of the fiber proposed in [5] can be observed in Fig. 1. When compared with the dispersion of our Proposal 1 in Fig. 5 one can notice that the total dispersion was displaced to 5 ps/nm·km (which helps to avoid non-linear effects) and that the wavelength range was more than doubled with a flatness difference of only 0.17 ps/nm·km.

In 2003, an IDF was designed in wavelengths of the C and L bands [13]. That paper describes very briefly the development carried out but omits characteristics of the refractive index profile which does not allow to simulate it nor visually show the total dispersion obtained to compare with our results.

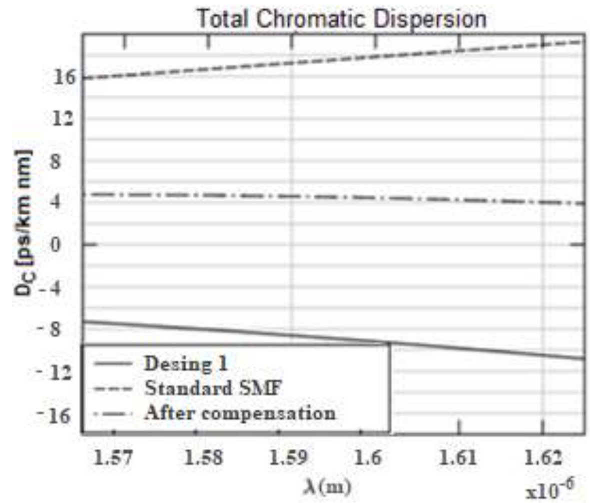

Fig. 6 Total Dispersion in the L band.

The SPPS method is known to be a numerical implementation that allows good precision for the calculation of the guided modes in an optical fiber. It allowed us to compute the characteristics of the considered fibers, mainly dispersion with low computing times because it involves simple operations (integrals). The method was verified as accurate and reliable reproducing results available in the literature for refractive index profiles previously analyzed with other methods.

The modification of the parameters of the refractive index profile allows to tailor the dispersion in the optical fiber. The effect of the modification of each parameter was analyzed on its role on the chromatic (waveguide + material) dispersion. We were able to propose designs that allowed us to manipulate the chromatic dispersion in the required way.

With the refractive index profiles of the Proposed Designs 1-3, dispersion curves inverse to those of a standard SMF in the wavelength range of 1400 nm to 1630 nm were achieved. Note that the curve obtained after compensation is almost flat but has a dispersion value different from zero. This considers non-linear effects that are not included in other works.

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