Improving Alias Rejection in Two-Stage Comb Decimation Filter

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***Abstract*—This paper presents novel multiplierless comb-based two-stage decimation structure. In the first comb stage is inserted the cascade of two simple multiplierless filters. The polyphase decomposition may be applied to the filters and thus all filtering is moved to lower rate. In the second stage is used sharpening of the comb section. The aliasing rejection is improved in all folding bands in comparison with the original comb. The method is illustrated with examples. Finally, the comparisons with methods from literature are provided.**

***Keywords*—decimation, aliasing, comb, sharpening, polyphase decomposition**

I. Introduction

Decreasing the sampling rate by an integer is called decimation [1]. This process consists of two operations: filtering and downsampling. Practically, the sampling rate is changed only by downsampling. However, since this operation introduces aliasing, it is necessary to filter input signal prior the downsampling to eliminate aliasing, which may deteriorate downsampled signal.

The most simple decimation filter is comb filter which has all coefficients equal to unity. Its transfer function is given as:

$$H(z)=\left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^K, \tag{1}$$

where M is the decimation factor and K is the number of cascaded combs, also called comb order.

The popular comb structure, called CIC (Cascaded-Integrator-Comb) filter is derived from (1). This structure is composed of cascaded integrators and combs separated by downsampling. The structure is very simple and does not require multipliers for its implementation. The CIC filter is an area efficient filter. However, the integrators working at high input rate make it a power inefficient filter.

Alternatively, the transfer function can be presented in a recursive form, and thus eliminate integrator section. As a result, a nonrecursive structure is a power efficient, but not area efficient.

Transfer function of comb in a nonrecursive form is given as:

$$H(z)=\left[\frac{1}{M}\sum_{k=0}^{M-1}z^{-k}\right]^K. \tag{2}$$

The polyphase decomposition is usually applied in a nonrecursive structure to move filtering to lower rate and thus decrease the power consumption.

Comb filter should attenuate the aliasing which occur in the bands around comb zeros, called folding bands.

The comb magnitude characteristic is given as:

$$\left|H(e^{j\omega})\right|=\left|\frac{1}{M}\frac{\sin(M\omega)}{\sin(\omega/2)}\right|^K. \tag{3}$$

As an example, Fig.1 presents comb magnitude characteristic for M=9 and K=1.

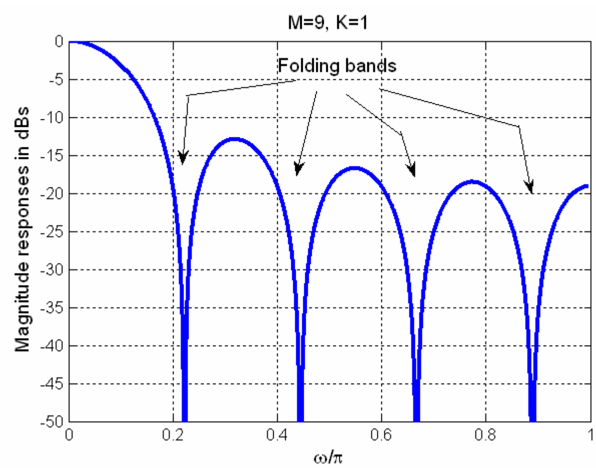


Fig.1. Magnitude response of comb filter.

However, the attenuation in comb folding bands is not high and should be increased. The simplest method is to increase the comb order K, but this will lead to the increase of the droop in the passband characteristic, which also may deteriorate the decimated signal.

Different method methods were proposed to increase comb aliasing rejection [2-8]. Since comb is a simple multiplierless filter, the methods for improving its alias rejection should be based on a multiplierless design.

This work presents new method for improving comb aliasing rejection based on sharpening and simple modified comb filters. The proposed filter is a multiplierless filter, and has an improved alias rejection in all comb folding bands.

The rest of the paper is presented in the following way. Second section presents a two-stage structure and its principal features. Section III introduces simple modified comb filters and sharpening technique. The proposed method is described and Section IV. The comparisons with methods from literature are given in Section V.

II. Two-Stage comb Structure

Consider that the decimation factor M can be presented as the product of two integers M_1 and M_2,

$$M = M_1 M_2 . \quad (4)$$

Comb transfer function (1) can be rewritten as:

$$H(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^K = \left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^K \left[\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right]^K = H_1(z)H_2(z^{M_1}) . \quad (5)$$

A two-stage structure decimated by M_1 and M_2, respectively, can be derived from (5). In the first stage is the comb filter $H_1(z)$, while in the second stage is the comb filter $H_2(z)$, where:

$$H_1(z) = \left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^K ; \; H_2(z) = \left[\frac{1}{M_2}\frac{1-z^{-M_2}}{1-z^{-1}}\right]^K . \quad (6)$$

Two-stage structure is presented in Fig.2.

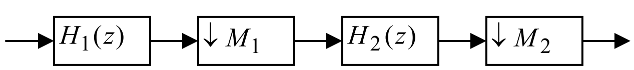


Fig.2. Two-stage comb structure.

Example 1: As an example Fig.3 shows overall magnitude responses of the comb with K=1 and M=20, and combs at first and second stages (6), taking M_1=4, M_2=5.

Note that the comb filter at the second stage contributes to comb alias rejection in the fifth and tenth folding bands. Generally, the filters in the second stage contributes to aliasing rejection in the comb folding bands which are multiples of M_2.

Similarly, the filters in the first stage contribute to comb aliasing rejection in all folding bands which are not multiples of M_2.

The same is confirmed in Fig.4, where are presented pole-zero plots for comb and combs at first and second stages, respectively.

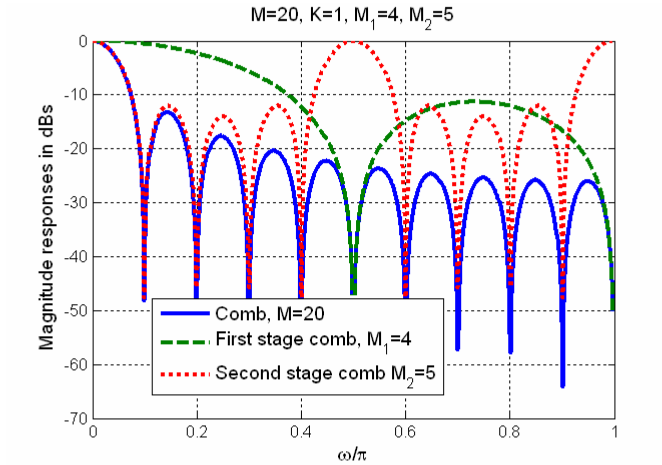
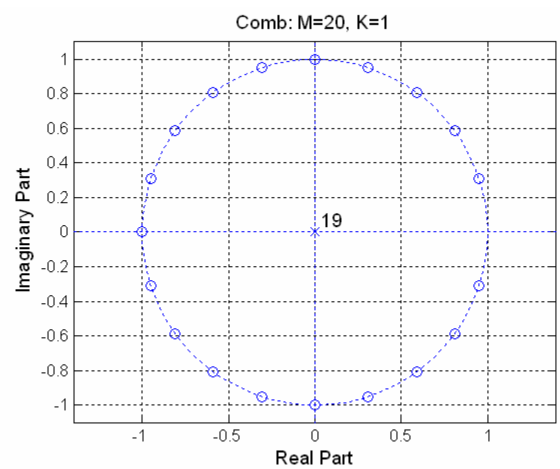


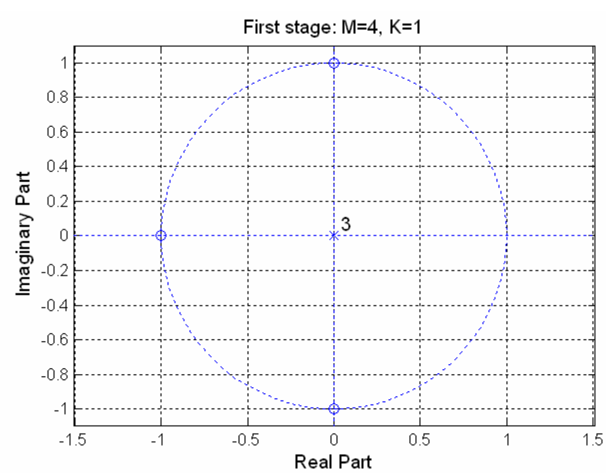
Fig.3. Magnitude responses of the overall comb and combs at first and second stages.

III. Modified Combs and Sharpening

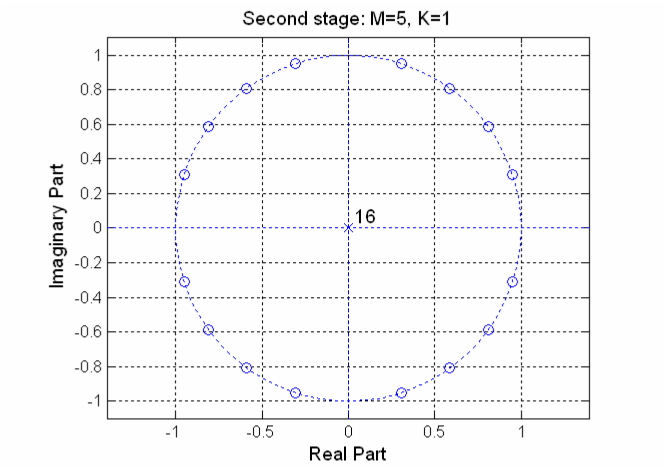
We propose here to use simple modified combs at the first stage and sharpening technique in the second stage.



a. Overall comb.



b. First stage.



c. Second stage.

Fig. 4. Pole-zero plots.

A. Modified combs

Consider the following modified comb filters derived from comb, by changing the middle coefficient.

$$G_1(z) = 1 + 0.5z^{-1} + z^{-2};\ G_2(z) = 1 + 1.5z^{-1} + z^{-2}\,. \qquad (7)$$

The cascade of filters (7),

$$G(z) = G_1(z)G_2(z)\,, \qquad (8)$$

is inserted into the first stage.

The filter (8) has all zeros on the unit circle, as shown in Fig.5. Those zeros increase aliasing rejection of comb in first stage.

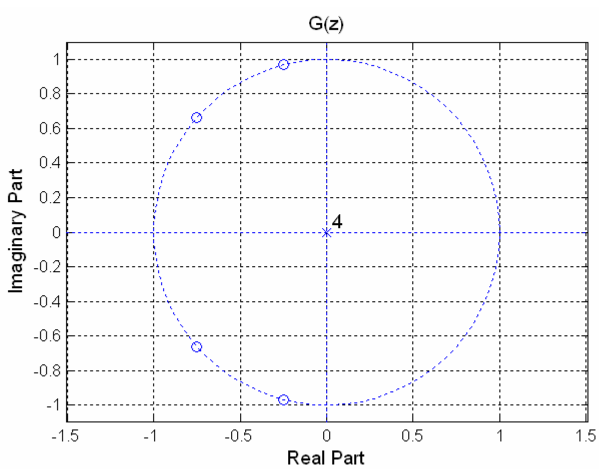


Fig. 5. Pole-zero plot.

Example 2: Fig. 6 illustrates how the filter (8) improves the aliasing rejection of the comb at the first stage.

B. Sharpening technique

The sharpening technique is introduced in [9] to simultaneously improve filter passband and stopband

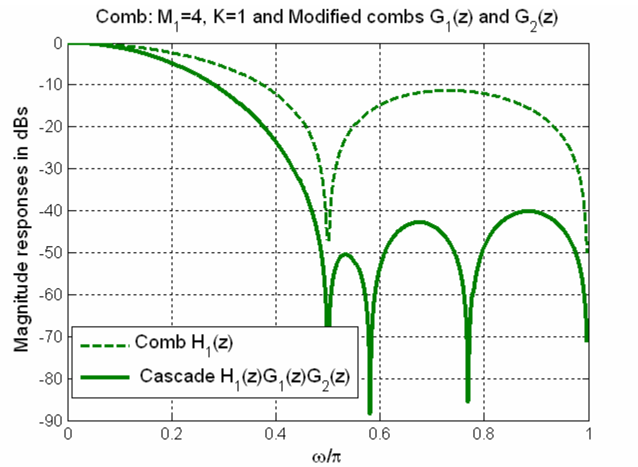


Fig. 6. Improving comb aliasing rejection at first stage.

We propose here to use the sharpening technique at the second stage using the sharpening polynomial $3X^2$-$2X^3$ [9]:

$$Sh\{H_2^{K_2}(z)\} = H_2^{2K_2}(3z^{-\lambda} - 2H_2^{K_2}(z))\,, \qquad (9)$$

where $Sh\{X\}$ means sharpening of X, and K_2 is the number of cascaded combs $H_2(z)$.

The delay λ is introduced to keep the linear phase of the sharpened filter (9). Consequently, the filter $H_2^{K_2}(z)$ must be odd.

IV. Proposed Filter

The proposed filter has two comb stages. First and second stages are decimated by M_1 and M_2, respectively. In the first stage is inserted filter (8), while in the second stage is performed sharpening (9).

A. Transfer function

Transfer function of proposed filter is given as:

$$H_p(z) = H_{p1}(z)H_{p2}(z^{M_1})\,, \qquad (10)$$

where $Hp_1(z)$ and $Hp_2(z)$ are filters at first and second stages, respectively:

$$H_{p1}(z) = H_1^K(z)G(z) = \left[\frac{1}{M_1}\frac{1 - z^{-M_1}}{1 - z^{-1}}\right]^K G(z)\,, \qquad (11)$$

where $G(z)$ is the filter given in (8).

$$H_{p2}(z) = H_2^K(z)Sh\{H_2^{K_2}(z)\} = \left[\frac{1}{M_2}\frac{1 - z^{-M_2}}{1 - z^{-1}}\right]^K Sh\{H_2^{K_2}(z)\}\,, \qquad (12)$$

where K_2 is the number of cascaded combs and $Sh\{x\}$ means sharpening x.

The method is illustrated in the following example.

Example 3. Consider M_1=3, M_2=5, and K_2=2. The comb has parameters M=15 and K=5. Magnitude responses are shown in Fig. 7.

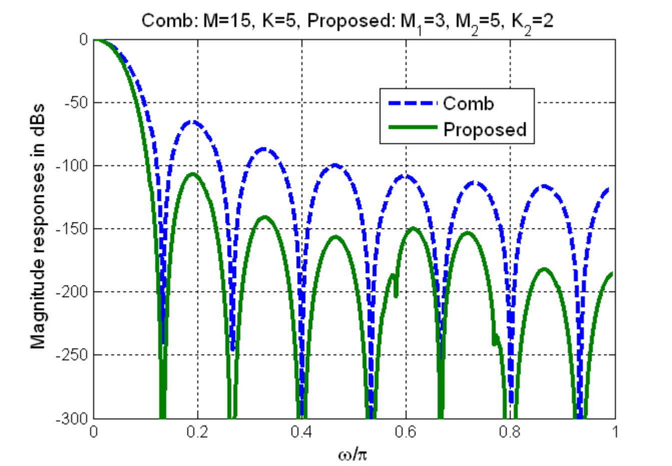


Fig. 7. Illustration of Example 3.

Observe that the aliasing rejection is improved in all folding bands in comparison with the comb filter.

In next subsection we discuss the choice of the design parameters.

B. *Choice of Design Parameters*

The design parameters are decimation factors M_1 and M_2 and comb parameter K_2.

The parameters M_1 and M_2 are chosen to be approximately equal, or M_1 is slightly less than M_2.

The choice of the parameter K_2 is related with the comb parameter K and the decimation factor M_2 in the following way:

For M_2 odd, $K_2 = \begin{cases} 2 & \text{for } \quad K = 5 \\ 1 & \text{for } \quad K = 1,2,3,4 \end{cases}$.

For M_2 even, K_2=2, for K=1,2,3,4,5. (13)

Note that this choice of K_2 will result in an odd $H_2^{K_2}(z)$.

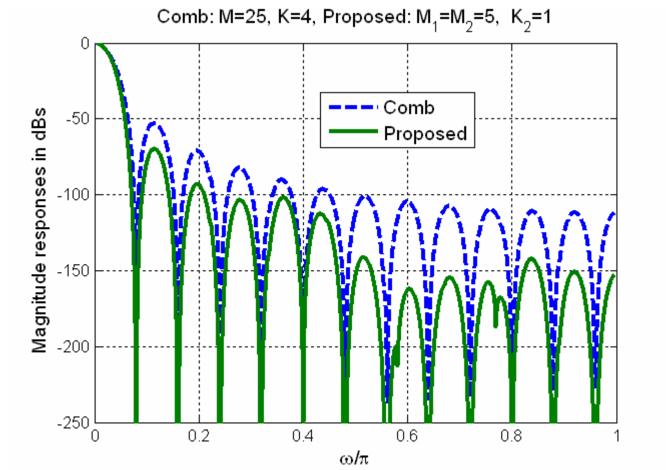
Example 4: In this example we illustrate the choice of parameters, considering two different values of M and K .

In the first case M=25, and K=4. The decimation factors are equal to M_1=M_2=5. From (13) we get K_2=1.

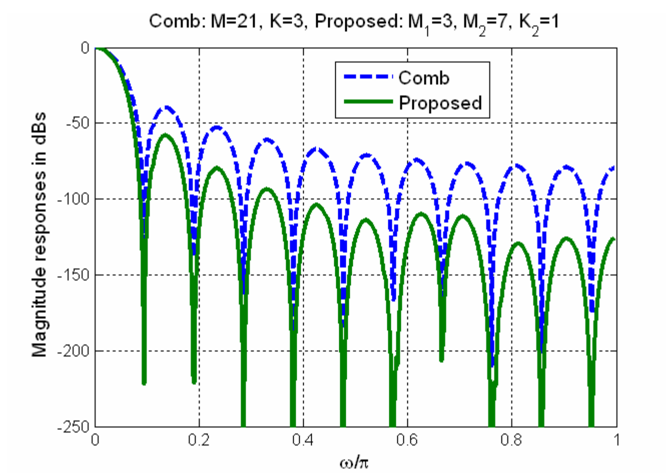
In the second case M=21 and K=3. The decimation factors are M_1=3 and M_2=7. From (13) we get K_2=3.

The magnitude responses of the proposed filter and the comb are contrasted in Fig.8. a and b. Note that the proposed filter provides higher aliasing rejection in all folding bands, in comparison with the comb filter.

From the other side, the proposed filter and comb have the similar passband characteristic.



a. M=26, K=4, M_1=M_2=5, K_2=1.



b. M=21, K=3, M_1=3, M_2=7, K_2=1.

Fig.8. Illustration of Example 4.

C. *Features of the Proposed Filter*

The principal features of the proposed filter are:

- There are only three design parameters: the decimation factors M_1 and M_2, and comb parameter K_2.
- The proposed filter is, like comb filter, a multiplierless filter.
- The proposed filter exhibits the similar magnitude characteristic in the passband as the comb filter. Consequently, the comb compensators may also be used for the proposed filter.
- The polyphase decomposition [1] can be applied to the filters in the first stage, and thus all filtering can be moved to lower rate i.e. after decimation by M_1.
- The only limit is that the decimation factor can be presented as a product of two integers.

V. COMPARISONS

In this Section we compare the proposed filter with the Methods in [4], [6], and [8].

A. Comparison with Sharpening Method [4]

In [4] is applied sharpening to the overall comb with the parameter *Ks*. For the sake of comparison we take M=20 and Ks=2. In the proposed method, the parameters are: M_1=4, M_2=5, K=5, and K_2=1. The magnitude responses are compared in Fig. 9.

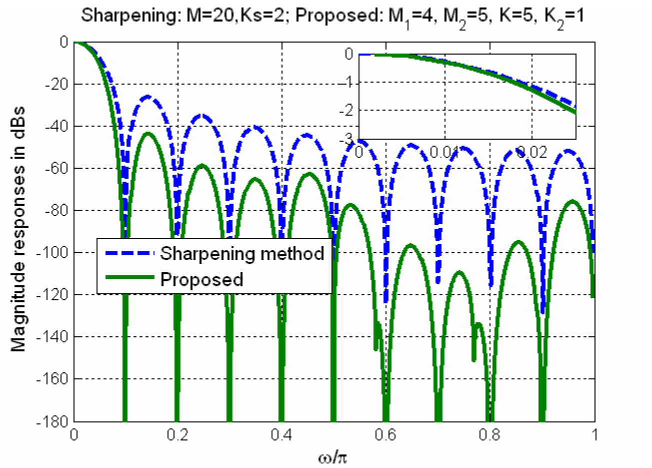


Fig. 9. Comparison with Method in [4].

B. Comparison with Method in [6]

The comparison is performed taking M=16. In Method [4] K=5 and the minimum attenuation is 100dB. In the proposed method the parameters of design are: M_1=M_2=4, K=5, K_2=2. The magnitude responses are contrasted in Fig.10. The proposed method exhibits higher attenuations in the folding bands. However, the advantage of the Method in [6] is the controlled minimum stopband attenuation.

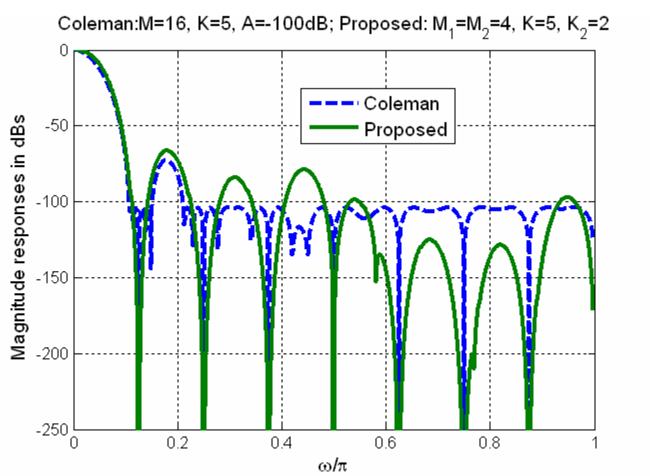


Fig. 10. Comparison with Method in [6].

C. Comparison with Method in [8]

. The methods are compared taking M=15 and presented in Fig.11, along with the design parameters.

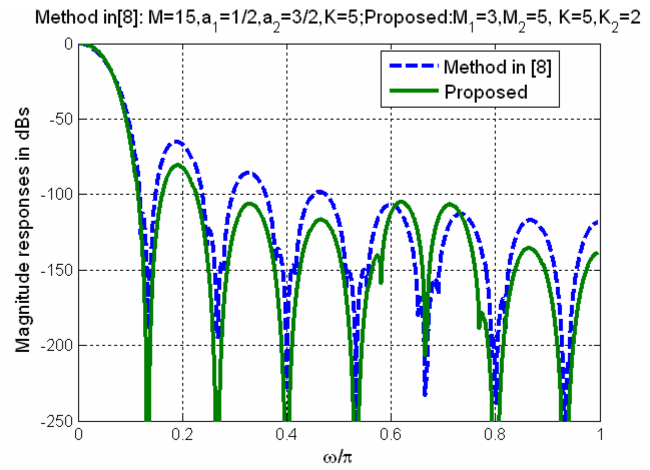


Fig. 11. Comparison with Method in [8].

VI. CONCLUSIONS

This paper presents novel multiplierless two-stage comb-based decimation filter with an improved alias rejection in all folding bands in comparison with the comb filter. Since the passband characteristics are similar, the comb compensator can also be used to compensate for the passband droop in the proposed filter. The parameters of design are the decimation factors and the order of comb in the second stage. The presented examples and comparisons show the benefit of the proposed filter.

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